Computer Methods in Applied Mechanics and Engineering 103 (1993) 231–246
North-Holland

CMA 395

# The use of computers in the design of discrete component systems

A.L. Schwab and K. van der Werff
*Department of Mechanical Engineering and Marine Technology, Delft University of Technology, Mekelweg 2, 2628 CD Delft, The Netherlands*

Received 17 September 1992
Revised manuscript received 14 December 1992

In the conceptual phase of mechanical engineering design, many design proposals are generated. The definite choice of promising designs is based on an evaluation of the performance of these concepts. For discrete component systems, part of this evaluation can be done by the finite element method, provided that this method is devised such that relatively large complex multi domain elements are available. Two examples are presented to illustrate the definition of such elements and the method of analysis.

## 1. Introduction

Mechanical engineering design is generally directed to a wide variety of systems and includes many different activities of the designer. In this contribution we shall confine ourselves to a special category of systems to be designed, the discrete component systems. Discrete component systems are systems which are composed of basic construction elements of various complexity, e.g. shafts, gearboxes, pumps. These construction elements are considered as black box elements. The properties of the construction elements are considered only as far as they are important to their functioning as a system component. We distinguish discrete component systems from continuous systems, like a water dam, for which the design task is devoted to an optimal or at least feasible geometry of the system.

Furthermore, we shall consider only a single phase in the design process, the preliminary design phase. Until now, CAD has been successful in the last stages of the design. The reason for this is that in this last phase of the design one can work quite formally according to well known procedures. In the preliminary design stages more creativity is needed and the approach is less formal. An important part of preliminary design activities is the evaluation of promising basic designs. The designer must be able to make a simple model of the future system. With this model the designer performs various calculations in order to determine whether the proposed system can meet the specifications. In order to do so the designer needs a modelling system with which he can easily define the future design. Furthermore, methods of analyses must be available for various types of systems.

*Correspondence to:* Dr. A.L. Schwab, Department of Mechanical Engineering and Marine Technology, Delft University of Technology, Mekelweg 2, 2628 CD Delft, The Netherlands.

## 2. The design process

In order to define the contents of our activities we shall briefly describe the design process. We restrict ourselves to a scheme described by Pahl and Beitz [1], which is characteristic for the continental approach to design theory. In this scheme we recognize five phases as shown in Fig. 1. Phase 1 is the clarification of the design task including further specification. Phase 2 is the phase of the conceptual design. In this phase the main problems are determined, the function structures are determined, principle solutions are generated, variants are combined and evaluated on technical and economical criteria. Phase 3 is the first embodiment design phase. Here the first layouts and form designs are produced. The best layouts are selected, improved and evaluated. In phase 4 the embodiment design is continued. The definition of form is completed, and possibly, optimizations are done, including checks for errors and cost, first Bill of Materials, Production Instructions. Phase 5 is the detail design phase in which detail drawings and production documents are made.

In this paper we shall concentrate on phase 2, the conceptual design phase. In a systematic design procedure function structures are sought which can basically fulfil the requirements. These function structures are abstract solutions for the design problem. The generation of possible function structures can be done by computer programs as this is a rather algebraic process, but by hand methods it is also possible to generate many alternatives. So the problem is not to generate solutions, it is to select from these the most promising. A senior designer uses his experience in order to generate only a few basic solutions. This can however hamper innovation. For this reason we would like to be able to do a quick materialisation of the function structure in order to evaluate the quality of a proposed design. This materialisation is effected by the definition of a composite of construction elements. For the various functions, a

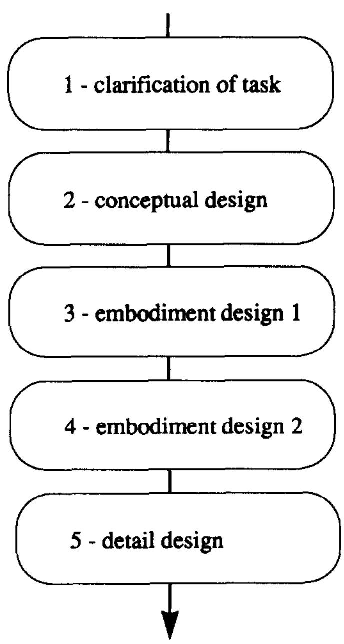



Fig. 1. The design process (from [1]).

number of materialisations are possible. The designer has at his disposal a library of construction elements with which he can synthesize his construction. By means of adequate computer programs he is able to perform the necessary capacity calculations of various kinds.

## 3. Modelling and analysis

Various methods are available for the modelling and analysis of the preliminary designs. Serious candidates are the so-called physical systems theory based methods like bondgraph methods. The idea is that for a number of elements we have a bondgraph available, maybe in a computer readable form. From these component bondgraphs a system bondgraph is assembled and consecutively analysed. Bondgraph methods have shown to be a very effective tool for the solutions of problems in various energy domains, e.g. mechanical, electrical and thermal.

Finite element methods have shown their capabilities in many engineering fields. Examples can be found in mechanics, kinematics, heat conduction, electrical conductance. Many codes are available for reasonable prices. The finite element method is usually considered as a method for the solution of continuum problems. The continuum is divided into finite elements of a relative simple form and equations can easily be assembled. A most important notion is the idea of 'divide and conquer' in order to master complicated systems. It is this idea that makes the finite element method especially fit for the modelling of discrete component systems.

The method to be used should meet the following requirements. In the first place it is required that the physical behaviour of the element can be described in terms of appropriate

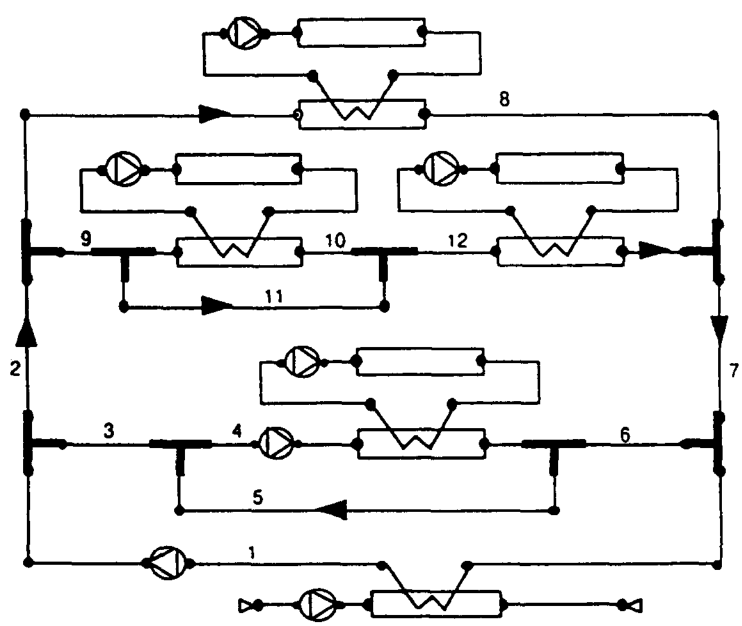



Fig. 2. Cooling system scheme of ships diesel engine (from [2]).

variables in the connecting nodes. Furthermore the element descriptions should obey the preservation laws of mass, energy and impulse. A model of a resistor, for example, should therefore contain the Joule effect $I^2R$, so that the temperature problem can be included easily. As many engineering problems are described in more than one energy domain, the model description and the analysis methods should be able to cope with these complex problems. An example of such a problem is the design of a cooling system for a ship diesel engine, see Fig. 2 [2]. In this problem the cooling effects are strongly coupled with the flow through the various components. It is not easy to describe this type of problems in such a way that in a single model all relevant aspects are covered. The last requirement is that the method of description is indeed a method and not an ad hoc system that is to be adapted for each new design type. This last requirement originates not only from an educational point of view but also from the need for efficient use of the designers capacity of understanding various different methods.

## 4. User interfaces

A most important aspect of the computer aided design system is the appearance to the designer, the user interface. The systems to be developed will be used interactively by mechanical designers. The way of using the systems will be that the designer picks construction elements from a menu and places them into the construction model. The systems must therefore not only be able to show the construction model on a screen, but they must also give possibilities to place the components in the model. This model is in general a three-dimensional model. The components generally have a three-dimensional extension. Because we want the systems to be highly interactive, high requirements are set on storage, representation and manipulation of the model. Regular 3D CAD-systems are not very useful because they are specially developed for the manipulation of complicated 3D-forms and they allow no access to their internals. It is however expected that analogous to the situation in electronic circuit design systems also for mechanical systems programs will be available in not too many years.

## 5. Examples

In the remainder of the paper we will give a brief exposition of a few pilot systems in which the different aspects have been studied. The system models were based on the finite element method. The finite element was chosen because this method fulfils most, but not all, requirements stated above in favour of bondgraph methods.

### *5.1. Example 1: gear transmission design project*

The gear transmission design project was chosen because, with only a few construction elements, it would be possible to demonstrate the ideas of the discrete component systems design. The construction elements used were a shaft element, a single gear and two types of bearings, axial-radial and radial only. The system consisted of a 3D-object editor, an interpreter/model generator and modules for kinematic, static and dynamic analysis.

With the 3D-object editor the designer defines type, number and position of the components of the gear transmission system. The 3D-object editor, which we called '123', has therefore the basic functions to create, select, modify and delete the components together with a number of viewing functions. In addition, combined functions were realized for efficient manipulating of the system. The presentation of the transmission system on the screen had to be done with a hidden line algorithm, even with simple designs. The definition of the system is meant to be like sketching, however some geometrical information must be precise. Basically, the 3D-object editor must therefore have the same capacities as a usual 3D CAD system. Our experience with the system was positive in a sense that we were able to define various gear transmission configurations. However, the time necessary to define even a simple transmission was too long. This aspect of the designer interface for this type of programs needs more attention. The result of the 3D-object editor session is a file consisting of the type and spatial position of the components. No information about connections between elements is defined.

The next step is an automatic interpretation and model building step. The objects and their positions form the input to a program that, based on a number of rules, generates a finite element model for the transmission. A typical rule in such a program could be: if two shafts have the same direction and if they have equal end point coordinates, within a certain precision, then the shafts are connected. For each component the program selects an appropriate finite element. Two meshed gears form together one gear pair element. When a gear happens to be mounted somewhere in the middle of a shaft then the shaft is automatically divided in two shaft elements. The program has been devised such that gear systems with non-parallel shafts are also correctly modelled. Examples are given in Figs. 3(a) and 4(a) [3]. The shafts are modelled as beam elements in a rotating coordinate system. Bearings are two node elements. Each node can be connected to different rotating elements. The choice for this type of description is mainly based on the wish to make a very clear interpretation of calculated stresses possible. The result of the second step is a file with the element model for the gear system. The following step is the kinematic analysis [3] of the model. The kinematic analysis is done for two reasons. The first reason is to see whether the basic design of the transmission system is as it is meant to be. The result of the kinematic analysis should show a

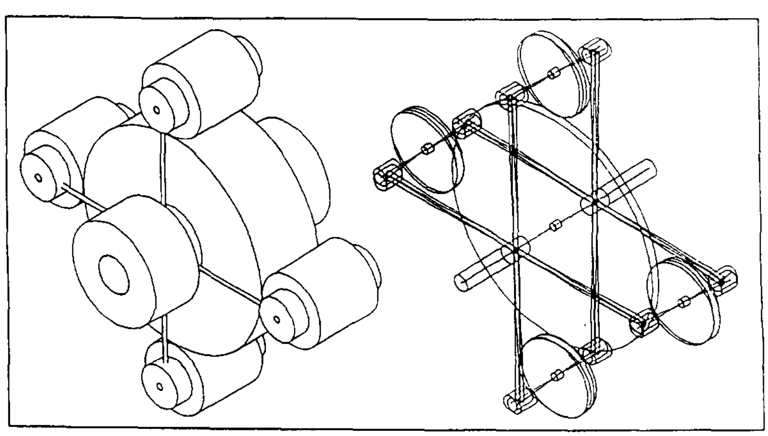


Fig. 3. Gear system model I and animated kinematic model (from [3]).

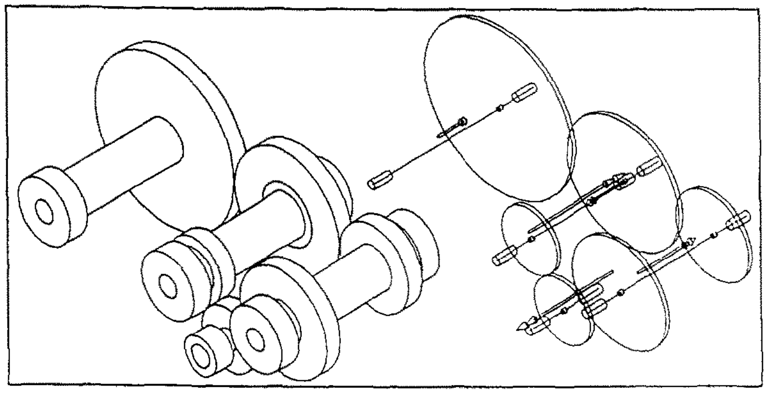


Fig. 4. Gear system model II and animated kinematic model (from [3]).

system with only the motions as they were intended by the designer. In this way we are able to warn the designer for inappropriate bearing systems, e.g. a missing axial bearing or unwanted static indeterminacy. The second reason is for checking the gear system's kinematic boundary conditions. If these are not correct, the following step of the static analysis will not be possible. The result of the kinematic analysis is presented graphically. The various motions are shown by animation, see Figs. 3(b) and 4(b) [3].

Before the static analysis can be carried out, the designer has to indicate the input shaft and the load of the gear system must be given. The load can be the working load, but also loads due to eccentric mass distributions or constant transverse forces on the shaft. If needed, the calculations can be repeated for a number of positions of the input shaft. Provisions have been made to analyse the time history of the resulting stresses in order to be able to make simple fatigue calculations as well. Also the designer has to input data about material properties and possibly data about stress concentration factors at diameter transitions. The output of the static analysis is again in graphic form: stress plots and also lifetime plots can be obtained. Of course the results are also available as a printed report. A problem with the presentation of the results is the fact that the results are results of a model that was generated by the computer. A back translation of the results to the original user model is therefore necessary.

For the designer it is very valuable to have the possibility of simple dynamic calculations. Apart from the static analysis the results of a dynamic analysis, albeit in a very simple form, can help the designer to avoid poor designs. A dynamic module was until now not integrated in the overall system. A preliminary system for torsional vibrations only has shown that this type of analysis gives valuable information to the designer. We recall that these calculations take place in the preliminary design phase, they are not the final design calculations. Figure 5 [4] shows some results for a real gear system. The figure shows the vibration energy in a sample five stage gear system due to a unit harmonic excitation at the input shaft. From the figure the designer may conclude that an operating speed corresponding to an excitation frequency of 370 (rad/sec) is dangerous.

### 5.2. Example 2: multi domain systems

In this project more complicated systems are modelled. We restrict ourselves to mechanic-hydraulic systems composed of the following components: electric motors, shafts, gearboxes,

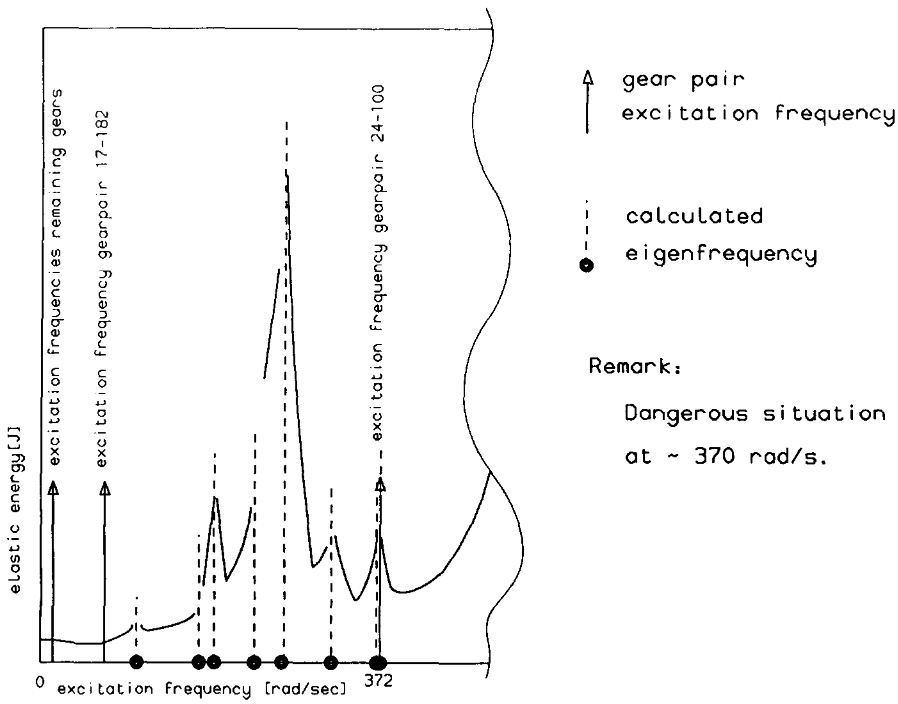



Fig. 5. Gear system frequency response (from [4]).

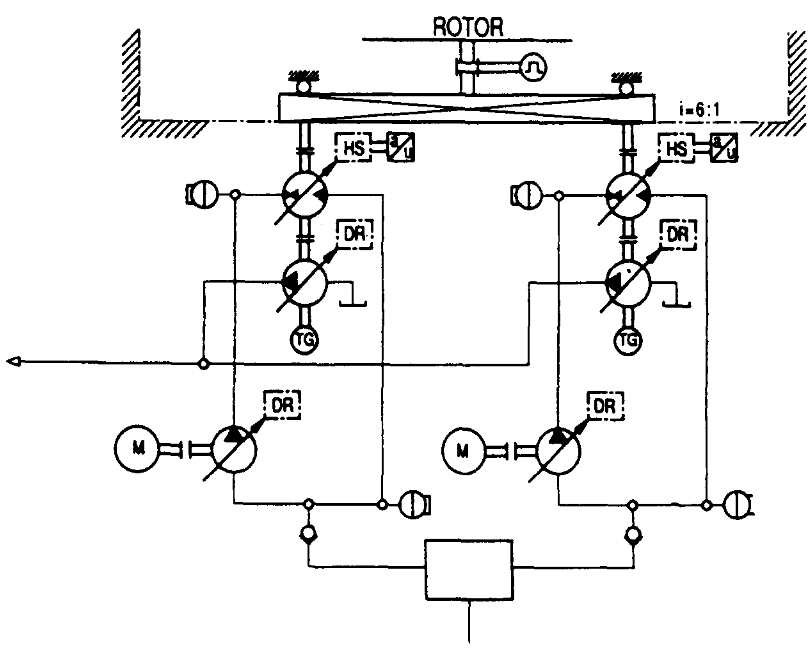



Fig. 6. An example of a hydraulic drive system for a geotechnical centrifuge.

pumps and pipes. The designer is interested in being able to evaluate the proper functioning of a proposed system including start–stop behaviour and possible off-design conditions. The computer model of the system must therefore include the complete characteristics of the various components. Especially the electric motor and the hydraulic pump characteristics need to be included. An example of the type of system to be investigated is shown in Fig. 6. The system represents the hydraulic drive system for a geotechnical centrifuge.

Traditionally, equilibrium states of this kind of systems are found by matching the characteristics of the driving and driven machines. In a simple system we have the matching of motor and pump drive characteristic, and the matching of the head discharge characteristic and the pipe resistance curve. In the case of more complicated systems there is no straightforward procedure. Again the problem can be solved by the finite element method. The class of problems to be covered is characterized more precisely by a description of the elements. The following elements will be presented: pipe elements, shafts, impeller pumps, squirrel cage motors and gearboxes. The flow is assumed to be incompressible, the shafts and gearboxes are modelled as undeformable elements.

The pipe element is a two node element (Fig. 7). The nodal variables are the pressure $p_i^e$ and the flow $Q_i^e$. For an element they are collected in the vectors $p^e$ and $Q^e$. Element variables are the pressure difference $\Delta p$ over the pipe and the flow $q$ through the pipe. The relation between nodal and element variables for element $e$ is given by

$$[\Delta p] = [-1 \quad 1]\begin{bmatrix} p_1 \\ p_2 \end{bmatrix}^e, \qquad \Delta p = D^e p^e , \tag{1}$$

$$\begin{bmatrix} -1 \\ 1 \end{bmatrix}[q] = \begin{bmatrix} Q_1 \\ Q_2 \end{bmatrix}^e, \qquad (D^e)^t q = Q^e . \tag{2}$$

In the nodes where pipe elements are connected they have the same nodal pressure $p_i$. This is expressed by

$$p^e = Cp . \tag{3}$$

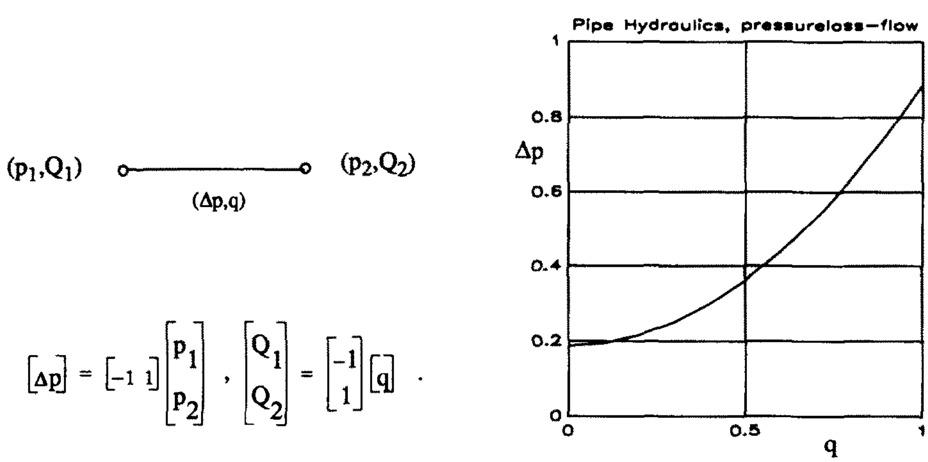



Fig. 7. Pipe element definition and characteristic.

Every row of the matrix $C$ has only one element equal to 1. The matrix $C$ identifies the nodal connections. Accordingly, the nodal flow $Q_i$ is the algebraic sum of the flow $Q_i^e$ of the pipes connected in that node,

$$C^t Q^e = Q . \tag{4}$$

The rows of $C^t$, being the columns of $C$, have just as many 1's as there are elements connected in the node corresponding to that row of $C^t$.

For the complete hydraulic system, the element and nodal variables have the following relation:

$$\Delta p = D^e C p , \tag{5}$$

$$(D^e C)^t q = Q . \tag{6}$$

The nodal flows $Q_i$ are usually zero for non-leaking pipe connections. This boundary condition together with (6) will later on be used for the derivation of an independent base for the description of the incompressible flow. In open systems, the nodal flow can be specified as a source or sink.

The characteristic of a pipe – the constitutive behaviour – is the relation between the pressure difference $\Delta p$ over, and the flow $q$ through the pipe element. In this study we considered only pressure loss, $-\Delta p$, due to turbulent flow,

$$-\Delta p = \frac{f}{d} \frac{1}{2} \rho q |q| \frac{1}{A^2} , \tag{7}$$

with the drag coefficient $f$, the pipe diameter $d$, the cross-sectional area $A$, the specific mass $\rho$ of the fluid and the element flow $q$.

The shaft element is a two node element. The node variables are the rotation speed $\omega_i^e$ and the torque $T_i^e$. Element variables are the torsion rate $\Delta\omega$ of the shaft and the torque $\sigma$ in the shaft. The relation between node and element variables for an element $e$ is given by

$$[\Delta\omega] = [-1 \quad 1] \begin{bmatrix} \omega_1 \\ \omega_2 \end{bmatrix}^e , \qquad \Delta\omega = D^e \omega^e , \tag{8}$$

$$\begin{bmatrix} -1 \\ 1 \end{bmatrix} [\sigma] = \begin{bmatrix} T_1 \\ T_2 \end{bmatrix}^e , \qquad (D^e)^t \sigma = T^e . \tag{9}$$

In the nodes where shaft elements are connected they have the same nodal rotation speed $\omega$. This is expressed by

$$\omega^e = C\omega , \tag{10}$$

similar to (3). Every row of the matrix $C$ has only one element equal to 1. The matrix $C$ identifies the nodal connections. Accordingly, the nodal torque $T_i$ is the algebraic sum of the torques of the shafts $T_i^e$ connected in that node,

$$C^{t}T^{e} = T \,. \tag{11}$$

For the complete mechanical system, the element and nodal variables have the following relation:

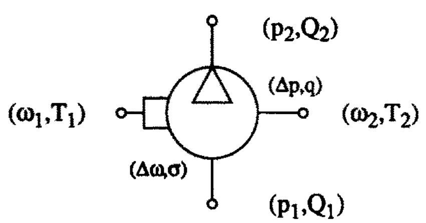
$$\Delta\omega = D^{e}C\omega \,, \tag{12}$$

$$(D^{e}C)^{t}\sigma = T \,. \tag{13}$$

In this study we considered all shafts as rigid, i.e. torsion rate equal to zero. From this condition an independent base for the description of rigid body motion can be derived. This will be shown later.

The remaining elements, the impeller pump, the squirrel cage motor and the gear box are power transformers. In the impeller pump mechanical power $\sigma\,\Delta\omega$ is transformed into hydraulic power $q\,\Delta p$.

The impeller pump is a four node element, see Fig. 8. Two nodes are mechanical: the first represents the input shaft of the pump, the second is a node connected with the pump frame. The latter is often fixed to the ground. The two remaining nodes are hydraulic. They represent the suction- and pressure-side of the pump. The element variables for the pump are the




$$[\Delta p] = [-1 \;\; 1]\begin{bmatrix} p_1 \\ p_2 \end{bmatrix}, \quad \begin{bmatrix} Q_1 \\ Q_2 \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \end{bmatrix}[q] \,.$$

$$[\Delta\omega] = [-1 \;\; 1]\begin{bmatrix} \omega_1 \\ \omega_2 \end{bmatrix}, \quad \begin{bmatrix} T_1 \\ T_2 \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \end{bmatrix}[\sigma] \,.$$

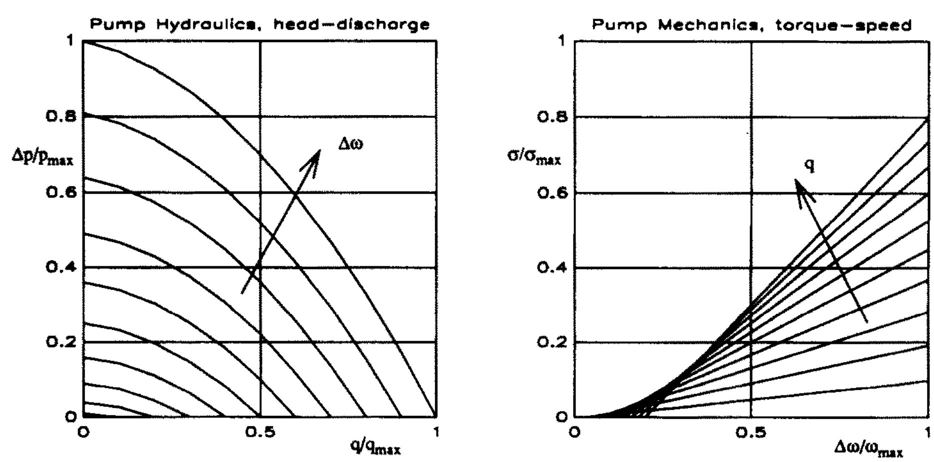



Fig. 8. Pump element definition and characteristic.

pressure difference $\Delta p$ (head) and the flow $q$ (discharge) through the pump, equations (1) and (2), and the difference in rotation speed $\Delta\omega$ between the shaft and the housing together with the torque $\sigma$ in the pump, equations (8) and (9). The power transformation is described in terms of constitutive relations $\Delta p = f_p(q, \Delta\omega)$ and $\sigma = f_\sigma(q, \Delta\omega)$. The pump characteristic as given by the pump manufacturer is usually a simple function $\Delta p = g(q)$ for the rated impeller speed. A reasonable model for the complete constitutive behaviour, see Fig. 8, is found realising that head is quadratic with impeller speed [5]. The pump torque $\sigma$ is found from a power balance using relatively simple assumptions for the power losses in the pump. In this study we assumed the constitutive pump behaviour as

$$\Delta p = \Delta p_t - \Delta p_1 , \tag{14}$$

with the theoretical head

$$\Delta p_t = p_{max}(\omega_s|\omega_s| - 2\kappa_2\omega_s q_s) , \tag{15}$$

and the head loss

$$\Delta p_1 = p_{max}(\kappa_3 q_s|q_s|) . \tag{16}$$

Here we have used the non-dimensional variables

$$\omega_s = \Delta\omega/\omega_{max} \quad \text{and} \quad q_s = q/q_{max} . \tag{17}$$

The maximum head at impeller speed $\omega_{max}$ is $p_{max}$. The discharge at which the head is zero for impeller speed $\omega_{max}$ is $q_{max}$. The coefficient $\kappa_2$, order of magnitude 0.1, characterizes the type of impeller pump. The coefficient $\kappa_3$ is a power loss coefficient. A reasonable value is $\kappa_3 = 1 + 2\kappa_2$. The pump torque is found from the power balance of an ideal pump,

$$\Delta p_t\, q = \Delta\omega\, \sigma . \tag{18}$$

Substitution of (15) results in the expression for the pump torque,

$$\sigma = \frac{p_{max}q_{max}}{\omega_{max}}\, q_s(|\omega_s| - 2\kappa_2 q_s) . \tag{19}$$

The power loss $\Delta p_1 q$ is transformed into heat which is not included in the model.

The electric motor is a four node element. Two nodes represent the connection to the electric mains. The two other nodes are mechanical, the first being the motor output shaft and the second is a node connected to the motor frame. The latter is often fixed to the ground. As we are not interested in the relatively fast electric transients a quasi static characteristic for a fixed supply voltage, the well known Klosz formula [6] was used. The definition of the element variables, motor torque $\sigma$ and relative rotation speed $\Delta\omega$, are according to (8) and (9). The constitutive behaviour according to Klosz is

$$\sigma_s = 2\,\frac{s_s}{1 + s_s^2} , \tag{20}$$

with the relative slip $s_s = s/s_k$, the slip $s = (\omega_s - \Delta\omega)/\omega_s$, the synchronous speed $\omega_s$, the breakdown slip $s_k$, the non-dimensional torque $\sigma_s = \sigma/\sigma_k$ and the breakdown torque $\sigma_k$.

Simple gearboxes are three node elements. Two nodes represent the input and the output shaft. The third node is a node connected with the gearbox casing which is normally fixed to the ground. The deformation rate connected with this element represents the kinematic relation between the two shafts and the casing,

$$[\Delta\omega] = [i \quad 1 \quad -(1+i)] \begin{bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{bmatrix}^e , \qquad \Delta\omega = D^e \omega^e . \tag{21}$$

The transmission ratio $i$ is equal to $-\omega_2/\omega_1$. This follows from (21) under the assumption of a rigid system, $\Delta\omega = 0$, and a fixed casing $\omega_3 = 0$. In the pilot study we assumed all gearboxes to be rigid. It will be obvious that the relation between the torque in the gearbox $\sigma$ and the nodal torques $T^e$ is

$$\begin{bmatrix} i \\ 1 \\ -(1+i) \end{bmatrix} [\sigma] = \begin{bmatrix} T_1 \\ T_2 \\ T_3 \end{bmatrix}^e , \qquad (D^e)^t \sigma = T^e . \tag{22}$$

The models of the power transformers described above have an interesting and useful property. Except for the usual behaviour they describe also the inversely driven systems, the pump can act as a turbine and the electric motor as a generator. The introduction of mechanical nodes describing the frame rotation of the pump, motor and gearbox facilitates, if such a node is fixed to the ground, the calculation of reaction torques. Interesting is the possibility to connect these nodes to moving nodes of the system. For instance, a normal gearbox can be transferred into a planetary gear system or a motor can be used for relative motion.

The use of the modelling system requires three steps: model definition, analysis and presentation of the results. The designer evaluates the results and repeats the process when necessary. The systems can be defined in a flow sheet manner. A menu of components is shown and the designer/engineer can pick components and place them on the screen. An example is shown in Fig. 9. When the definition is finished, the computer determines how the various system components are interconnected. This is done by a simple rule; when component nodes are near, they are connected in a system node. Some provisions have been made so that for example a pipe cannot be connected to an electric motor. The system equations are generated automatically. Basically we have two sets of system equations, one describing the flow rates, the other the rotation accelerations. The equations are first order ordinary differential equations. First order because inertia effects of the flow and the mechanical motion are described in terms of speeds. We are not interested in the position of fluid parts or of shafts. As the pilot system was programmed in MATLAB [7] we had a rich palette of linear algebra procedures at our disposal. This encouraged us to describe the system equations as follows.

First an independent base for the flow is calculated. We start with (6) which already describes incompressibility. The boundary conditions on most non-leaking pipe connections

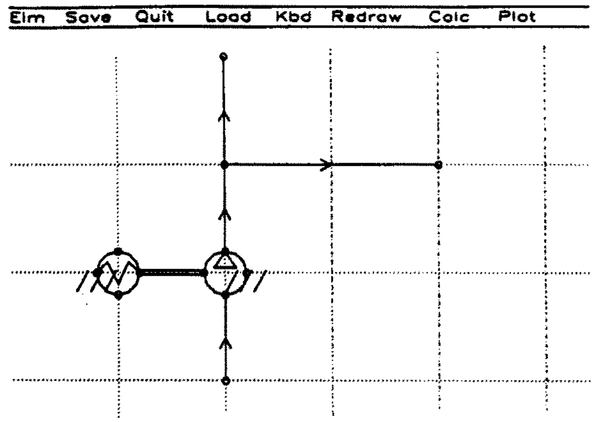



Fig. 9. Flowsheet pump system.

are expressed such that a subset of $Q$, $Q^0$ must be zero. From these we derive an independent base $q^*$ for the element flows $q$,

$$Q^0 = ((D^e C)^t)^0 q = 0 \quad \Rightarrow \quad q = Bq^* . \tag{23}$$

The equilibrium state of the system can be found by numerical integration of the dynamic equilibrium equations, i.e. the equilibrium equations extended with inertia forces. For a single pipe element we have one equilibrium equation (1) expressing the stationary equilibrium, $\dot{q} = 0$. The inertia term is found from the following consideration. Application of Newton's second law on a thin fluid segment of cross-section A and integration over the pipe length leads to

$$\int_1^2 \frac{\mathrm{D}v}{\mathrm{D}t} \rho A \,\mathrm{d}x = -\int_1^2 A \,\mathrm{d}p , \tag{24}$$

with the material acceleration

$$\frac{\mathrm{D}v}{\mathrm{D}t} = \dot{v} + v \frac{\partial v}{\partial x} , \tag{25}$$

or in terms of the flow $q = vA$,

$$\frac{\mathrm{D}v}{\mathrm{D}t} = \frac{1}{A}\left(\dot{q} - \left(\frac{q}{A}\right)^2 \frac{\partial A}{\partial x}\right) . \tag{26}$$

This leads to the dynamic equilibrium equation for an element,

$$p_2 - p_1 + \frac{1}{2}\rho\left(\frac{1}{A_2^2} - \frac{1}{A_1^2}\right)q^2 + \rho \int_1^2 \frac{\mathrm{d}x}{A} \dot{q} = 0 . \tag{27}$$

For $\dot{q} = 0$, (27) is recognized as the Bernoulli equation for incompressible stationary flow.

The dynamic extension of the static equilibrium equation (1) is

$$\Delta p = D^e p^e + \frac{1}{2}\rho\left(\frac{1}{A_2^2} - \frac{1}{A_1^2}\right)q^2 + \rho\int_1^2 \frac{\mathrm{d}x}{A}\,\dot{q}\,. \tag{28}$$

For an element with constant cross-sectional area $A_1 = A_2 = A$ the dynamic equilibrium becomes

$$\Delta p = D^e p^e + M^e \dot{q}\,, \quad \text{with } M^e = \frac{\rho l}{A}\,. \tag{29}$$

The equations for a complete system composed of these kind of elements are

$$\Delta p = Dp + M\dot{q}\,, \quad \text{with } D = D^e C \text{ and } M = \mathrm{diag}(M^e)\,. \tag{30}$$

Next we transform (30) to the independent base $q^*$ using (23), leading to the dynamic equilibrium equations in terms of the new base,

$$(B^t M B)\dot{q}^* = B^t(\Delta p - Dp)\,. \tag{31}$$

Note that the linear mapping $B^t MB$ is nonsingular due to the independent base. Equations (31) together with the constitutive equations $\Delta p = f(q)$ for each element are used for numerical integration. The equilibrium state of the system is found by integrating up until vanishing $\dot{q}^*$.

For the mechanical part of the system we will follow the same strategy. First an independent base $\omega^*$ for the nodal rotation speeds $\omega$ is calculated from the assumption that most elements, i.e. shafts and gearboxes are rigid, $\Delta\omega^0 = 0$, hence

$$(D^e C)^0 \omega = 0 \quad \Rightarrow \quad \omega = Z\omega^*\,. \tag{32}$$

The mechanical equilibrium is extended with inertia forces resulting in the dynamic equilibrium equations,

$$T = D^t\sigma + M\dot{\omega}\,, \quad \text{with } D^t = (D^e C)^t \text{ and } M = \mathrm{diag}(J^e)\,, \tag{33}$$

with $J^e$ being the lumped rotational inertia for each element.

Next, (33) is transformed to the base $\omega^*$ with (32). This leads to the dynamic equilibrium equations in terms of the independent base $\omega^*$,

$$(Z^t M Z)\dot{\omega}^* = Z^t(T - D^t\sigma)\,. \tag{34}$$

Note that the linear mapping $Z^t MZ$ is non-singular due to the independent base. Together with the constitutive equations $\sigma = f(\Delta\omega)$ for each non-rigid element, equations (34) are ready for numeric integration. The equilibrium state of the system is found by integrating until vanishing $\dot{\omega}^*$.

For the numerical integration in MATLAB we used a special FORTRAN integration

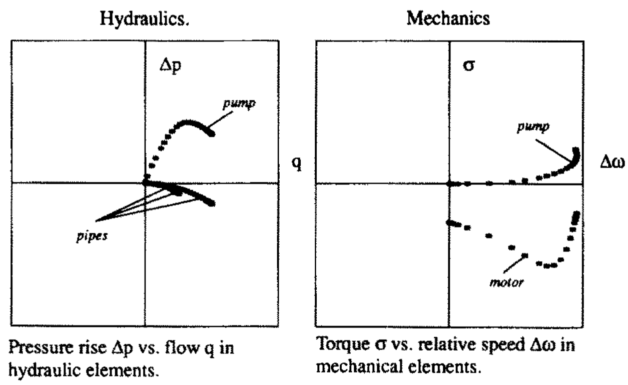



Fig. 10. Pump system startup behaviour.

procedure based on a modified divided difference form of the Adam's predictor corrector formulas as implemented by Shampine and Gordon [8]. We have very good experience with this method especially because it uses few function-evaluations and it has automatic selection of order and stepsize.

On first sight it seems that the descriptions in each domain are independent. However, the system variables of the various domains are coupled by the constitutive laws of the transformer elements like pumps and motors. As an example we consider the role of a pump. We need to know both the flow $q$ through the pump and the relative impeller speed $\Delta\omega$ in order to calculate the head $\Delta p$ and pump torque $\sigma$. The interconnection is found on the constitutive element level. Therefore both dynamic equilibrium equations (31) and (34) have to be integrated simultaneously.

The results of the analyses are presented in a customary engineering form. During the calculation of the start-up behaviour, the states of pipes, pumps and motors are continuously plotted in the characteristic graphs; an example is shown in Fig. 10. Results, such as pressure distribution and rotation speeds, are presented only for the steady state situation in the flow sheet.

## 6. Conclusion

This article does not give the full and ultimate solution of the problem stated. The modelling of real engineering systems is far from trivial and we want to stimulate experts in FEM methods to give attention to this kind of problems. Still unsatisfactory is the inclusion of heat generation and transport in the model. Further, the models as presented do not fulfil our wish for energy conservation. The elements described are still energy sources and sinks. Including heat dissipation into the models for machine components turns them into hot items.

## References

[1] G. Pahl and W. Beitz, Konstruktionslehre (Springer, Berlin, 1977).

[2] R.G. Seidl, COOSEL, Master Thesis, Delft University of Technology, Faculty of Mechanical Engineering and Marine Technology, 1991.

[3] A.L. Sytstra, Kinematic analysis of transmissions, based on the FEM, in: AGMA Fall Technical Meeting (Toronto, 1990) 1–15.
[4] G.C. Avontuur, Dynamic calculations for transmissions, Master Thesis (in Dutch), Delft University of Technology, Faculty of Mechanical Engineering and Marine Technology, 1992.
[5] S. Lazarkiewicz and A.T. Troskolanski, Impeller Pumps (Pergamon, Oxford, 1965).
[6] M. Klosz, Drehmoment und Schlüpfung des Drehstrommotors, Arch. Electrotech. 5 (1916) 59–87.
[7] PC-MATLAB User's Guide (The Mathworks Inc., South Natick, 1989).
[8] L.F. Shampine and M.K. Gordon, Computer Solution of Ordinary Differential Equations (Freeman, San Francisco, 1975).