Proceedings of the ASME 2008 International Design Engineering Technical Conferences & Computers and Information in Engineering Conference
IDETC/CIE 2008
August 3-6, 2008, Brooklyn, New York, USA

DETC2008-49400

ON THE INFLUENCE OF CONTACT GEOMETRY ON GRASP STABILITY

Gert A. Kragten[1*], **Just L. Herder**[1], **A. L. Schwab**[2]
Delft University of Technology
1) Department of BioMechanical Engineering (BMechE)
2) Laboratory for Engineering Mechanics
Mekelweg 2, 2628 CD Delft, The Netherlands
Email: g.a.kragten@tudelft.nl

ABSTRACT

This paper demonstrates that the predicted grasp stability is highly sensitive to only small changes in the character *of the contact forces. The contribution of the geometry and stiffness at the contact points to the grasp stability is investigated by a planar grasp with three contact points. Limit cases of zero and infinite contact curvatures, and finite to infinite contact stiffnesses are considered. The stability is predicted based on the approach of Howard and Kumar [1], and verified with multibody dynamic simulations. For rigid objects and fingers with only normal contact stiffness, the grasp stability is dominated by the contact geometry, whereas the local contact stiffness and preload have a minor effect. Furthermore, grasps with pointed finger tips are more likely to be stable than grasps with flat finger tips.*

INTRODUCTION

The stability of a grasped object is a commonly used property to assess grasp performance. Predicting stability is relevant both at the grasper design stage to determine for instance the optimal shape of the fingers, and in the planning of the grasper to put the fingers at spots on the object that will result in a stable grasp. Stability is a dynamic phenomenon: it considers the behavior after small variations about the equilibrium state of a grasped object. Mathematically, the tangent stiffness matrix (**K**) in the linearized equations of motion of a grasped object about the equilibrium state must be positive definite for stability. This **K** matrix is derived from force and moment terms that are differentiated with respect to the position and orientation of the grasped object. These terms, representing the *character* of the grasp forces, must therefore be well addressed in order to understand and accurately predict the grasp stability.

Forces are defined by a point of application, magnitude and a line of action. For grasping, the change of these three aspects caused by small variations of the equilibrium configuration depends on the contact model between grasped object and fingers. This contact model can depend on several physical effects like the local material stiffness, the geometry at the contact points, the finger kinematics, and the actuation of the fingers. In literature, many different contact models are presented, varying between normal linear elastic point contact (*e.g.* [2]) to for example a finite nonlinear deformable contact area [3]. However, the validity of the assumptions on which these models are based, is often not demonstrated. Therefore it is unknown whether or not the predicted stability resembles reality.

The objective of this paper is to demonstrate the strong dependency of the predicted object stability on the character of the contact forces by means of an example. We will consider a planar grasp with three contact points, and evaluate all limit cases, which are combinations of zero and infinite contact curvatures. In addition, we will vary the normal contact stiffness from finite to infinite (constraints) for each of the three contact points. We will be using two methods. Firstly, the stability is predicted by the frictionless contact force model presented by Howard and Kumar [1]. Secondly, these results are verified by a multibody

[*] Address all correspondence to this author.

dynamics simulation. Based on this study, we will determine the relative contributions to the grasp stability of the contact geometry, the contact stiffness and the preload. Finally, the implications of the results on the design of grasping devices will be discussed.

The study in this paper was motivated by the observation that the predicted stability of a grasp can invert from stable to unstable, when the character of the contact forces is, seemingly, changed only slightly. Application of a graphical method to determine stability from [4] to the grasp examples from [1], resulted in totally different results in the predicted stability. This could only be attributed to the differences in the contact geometry and local stiffness. To clearly demonstrate these effects on the stability, this paper uses an existing contact force model where friction is ignored.

MATERIAL AND METHODS

Grasp example

To investigate the sensitivity of the grasp stability on the contact geometry and stiffness, a planar grasp example with three contact points is used. This example was introduced by Howard and Kumar, and consists of three rigid fingers grasping a rigid polygon with dimensions corresponding to Fig. 1. According to their assumptions, the fingers are rigidly fixed in the global reference frame O, representing position controlled fingers. To each ith finger a local coordinate frame (x_i, y_i) is fixed. Furthermore, friction at the contact points is neglected, and the object can freely move by what is called 'rigid body penetration'. The latter assumption sounds confusing, but means that for small object movements the fingers or object can indent without changing the initial contact geometry. The stability of the object is expressed by the distance r between finger 3 and frame O at which the third contact force must apply in order to stably grasp the object.

Contact Force Model for Curved Surfaces

The derivation of the **K** matrix based on the contact force model of Howard and Kumar is briefly recapitulated. It is assumed that only normal contact forces apply, and that the local contact geometry of the fingers and object are sections of circles with known radii. In Fig. 2 such a contact point is shown, with the local orthogonal coordinate frame (O_i) fixed to the finger. In addition, three surface parameters are used: u_A and u_B along the finger and object, respectively, and the rigid body penetration w perpendicular to the contact surface. The local frame is translated by (d_{x_i}, d_{y_i}) and rotated by Φ_i with respect to the global frame (O). K_A and K_B are the curvatures (inverse of radii) of the finger and object, respectively.

For small displacements of the object, the magnitude of the contact force will change according to:

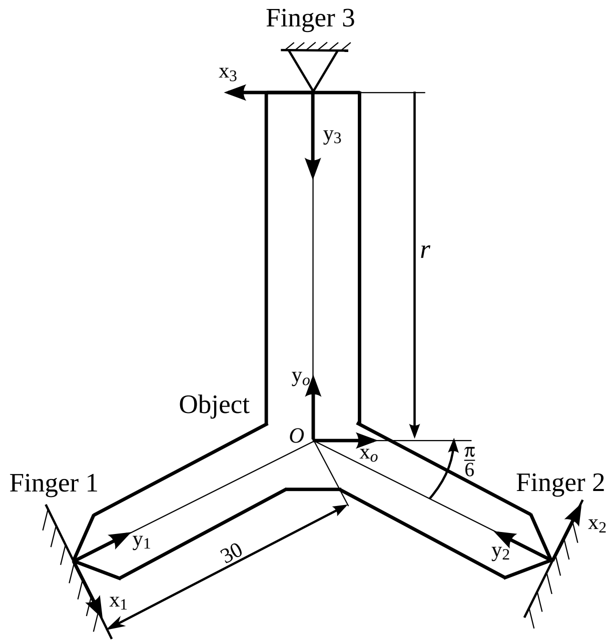

Figure 1. Grasp of a polygon by rigid fingers with zero (Finger 1 and 2) and infinite (Finger 3) curvature, adapted from [1]. Dimensions in [mm], [rad].

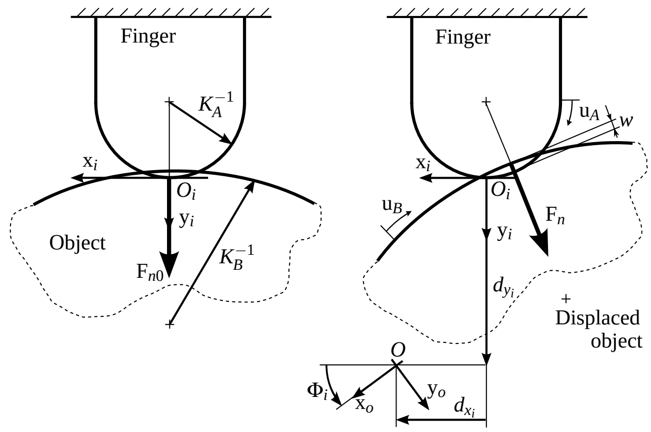

Figure 2. Local surface frame and orthogonal frame at a contact point, adapted from [1].

$$F_n = F_{n0} + k_n \Delta w \tag{1}$$

where F_{n0} is the initial contact force and k_n the normal contact stiffness. The linearized variation of the contact forces and moment at O_i, expressed in the surface parameters is:

$$\begin{aligned} \Delta F_{x_i} &= -F_n \sin(-K_A \Delta u_A) \approx F_{n0} K_A \Delta u_A \\ \Delta F_{y_i} &= F_n \cos(-K_A \Delta u_A) - F_{n0} \approx k_n \Delta w \\ \Delta M_{O_i} &= -F_n \frac{\sin(-K_A \Delta u_A)}{K_A} \approx F_{n0} \Delta u_A \end{aligned} \tag{2}$$

The transformation from the surface parameters u_A, u_B and w to the local frame is obtained by assuming superposition of pure slipping, rolling and indentation of the object at the finger surface. The first two types of these motions are visualized in Fig. 3. On the left part, the object purely slips a horizontal distance Δx_i. On the right side of this figure, the object purely rolls by an angle $\Delta\theta_i$. Based on geometry, the following relations hold:

$$\begin{aligned} \Delta u_A &= \frac{\Delta x_i K_B}{K_A + K_B} - \frac{\Delta\theta_i}{K_A + K_B} \\ \Delta u_B &= \frac{\Delta x_i K_A}{K_A + K_B} + \frac{\Delta\theta_i}{K_A + K_B} \\ \Delta w &= -\Delta y_i \end{aligned} \tag{3}$$

Substituting Eqn. (3) in Eqn. (2) and rearranging terms leads to the following relation:

$$\begin{pmatrix} \Delta F_{x_i} \\ \Delta F_{y_i} \\ \Delta M_{O_i} \end{pmatrix} = -\mathbf{K}_i \begin{pmatrix} \Delta x_i \\ \Delta y_i \\ \Delta\theta_i \end{pmatrix} \tag{4}$$

where $\mathbf{K}_i$ is the local tangent stiffness matrix, consistent with Eqn. (20) in [1]:

$$\mathbf{K}_i = \begin{bmatrix} \frac{-F_{n0} K_A K_B}{K_A + K_B} & 0 & \frac{F_{n0} K_A}{K_A + K_B} \\ 0 & k_n & 0 \\ \frac{-F_{n0} K_B}{K_A + K_B} & 0 & \frac{F_{n0}}{K_A + K_B} \end{bmatrix} \tag{5}$$

To obtain the overall tangent contact stiffness matrix expressed in the global frame, the local $\mathbf{K}_i$ matrix of each contact point i is transformed and added:

$$\mathbf{K}_{tot} = \sum_{i=1}^{n} \mathbf{T}_i^T \mathbf{K}_i \mathbf{T}_i \tag{6}$$

where ^{T} means transpose and $\mathbf{T}_i$ is the transformation matrix:

$$\mathbf{T}_i = \begin{bmatrix} \cos(\Phi_i) & -\sin(\Phi_i) & d_{y_i} \\ \sin(\Phi_i) & \cos(\Phi_i) & -d_{x_i} \\ 0 & 0 & 1 \end{bmatrix} \tag{7}$$

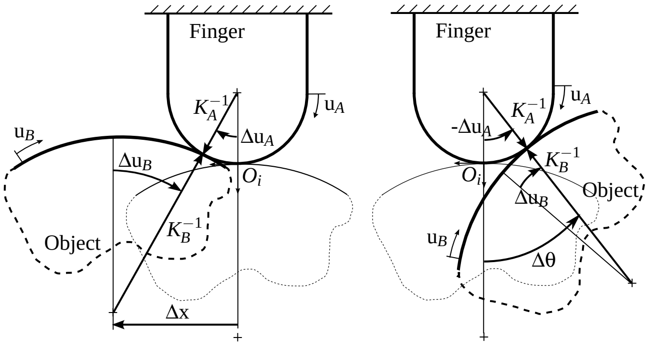

Figure 3. Relative motion of the object by slip (left) and roll (right).

with d_{x_i}, d_{y_i} and Φ_i according Fig. 2, and n is the number of the contact point. For stability, $\mathbf{K}_{tot}$ must be positive definite, which in this case is satisfied when the determinant of $\mathbf{K}_{tot}$ is positive.

Contact Geometry and Stability. For the planar grasp example (see Fig. 1), the stability is evaluated for all limit cases of zero and infinite contact curvatures. A finger with zero curvature is called *flat*, for which contact point $K_A = 0$, $K_B \to \infty$. Fingers with infinite curvature are called *pointed* and for such contacts $K_A \to \infty$, $K_B = 0$.

Since there are three fingers which are either pointed or flat, 2^3 different combinations exist as shown in Fig. 4. For each combination, the distance r for which the object is stably grasped is calculated. Taking case *ffp* as example to calculate for which r this grasp is stable, $K_{A_1}, K_{A_2}, K_{B_3} = 0$ and $K_{B_1}, K_{B_2}, K_{A_3} \to \infty$ are substituted into Eqn. (6), which results in:

$$\mathbf{K}_{tot,\text{ffp}} = \begin{bmatrix} \frac{3(k_{n,1}+k_{n,2})}{4} & \frac{\sqrt{3}(k_{n,1}-k_{n,2})}{4} & -F_n \\ \frac{\sqrt{3}(k_{n,1}-k_{n,2})}{4} & \frac{k_{n,1}+k_{n,2}+4k_{n,3}}{4} & 0 \\ -F_n & 0 & F_n(r-60) \end{bmatrix} \tag{8}$$

The determinant of this matrix (which must be positive for a stable grasp) is as follows:

$$\det\left[\mathbf{K}_{tot,\text{ffp}}\right] = F_n\,(3r-180)\,\frac{k_{n,1}k_{n,2} + k_{n,1}k_{n,3} + k_{n,2}k_{n,3}}{4} - \frac{F_n^2\,(k_{n,1} + k_{n,2} + 4k_{n,3})}{4} \tag{9}$$

For stability, the magnitude of distance r for which Eqn. (9) is positive is derived.

Contact Stiffness and Stability. Howard and Kumar initially assumed infinitely stiff contact for each finger ($k_n \to \infty$). However, this assumption results in grasps without any degree of freedom from a kinematic viewpoint. To investigate the effect

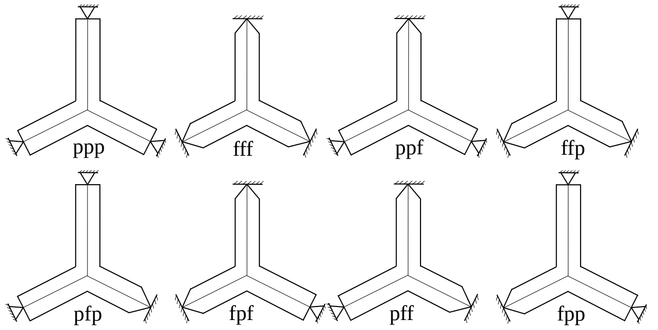

Figure 4. Limit cases of zero and infinite contact curvatures. Letter p denotes *pointed* finger, f denotes *flat* finger tip, and the ith letter belongs to the ith finger.

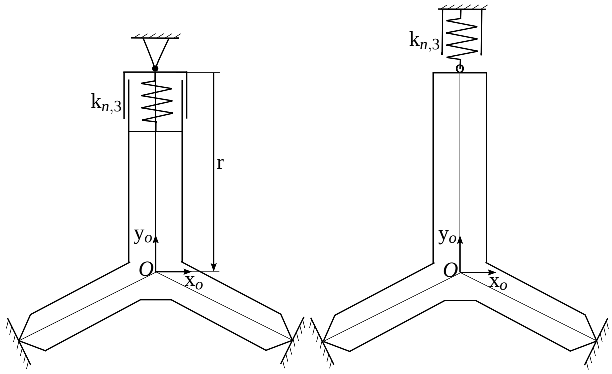

Figure 5. Grasp with finite stiffness at contact 3 in the object (left) and in the finger (right), respectively.

of the contact stiffness on the stability, the local stiffness matrix (Eqn. (5)) is evaluated for all eight combinations of pointed and flat finger tips, both with infinite and finite normal contact stiffness $k_{n,i}$ in each ith contact point. For this symmetric example to be in equilibrium, the preload F_{n0} at each finger must be equal. Based on the obtained stability conditions as functions of $k_{n,i}$ and F_{n0}, the contribution of the contact stiffness to the stability will be discussed.

Multibody Dynamics Simulation

To verify the predicted stability in the various cases according to Howard and Kumar, a flexible multibody dynamics model was made. The modeling was done in the program system SPACAR which was developed by Van der Werff [5], Jonker [6], and Schwab and Meijaard [7]. SPACAR is based on finite element principles and can handle systems of rigid and flexible bodies connected by various joints in both open and closed kinematic loops. SPACAR numerically generates and solves full non-linear dynamics equations using minimal coordinates (constraints are eliminated). SPACAR can also find the numeric coefficients for the linearized equations of motion based on an analytic linearization of the non-linear equations. This option was used for determining the total stiffness matrix $\mathbf{K}_{tot}$ and the corresponding stability.

The initial assumption of rigidly fixed fingers and infinitely stiff fingers kinematically results in a grasp without any degree of freedom. Therefore at least one contact point must have finite stiffness in order to allow for small variations about the equilibrium state and to predict stability in the multibody dynamics model. First, one linear elastic spring element (arbitrarily chosen $k_n = 30$ Nmm^{-1}, $F_{n0} = 3$ N) is applied in subsequently the finger and the object at contact 3 and 1, while the stiffness of the other contact points is kept infinite (by constraints). The modeling of a spring element at contact 3 in the object and finger, respectively, is illustrated in Fig. 5. The principal difference between the first and the latter is whether the spring element rotates during small variations of the object or not. Secondly, for the case with the flat-flat-pointed finger shape (see Fig. 1) also two and three fingers with finite stiffness were simulated, which resulted in grasps with two and three degrees of freedom, respectively. For all these simulated cases, distance r for which the grasp is stable was calculated and compared with the results obtained with the contact force model of Howard and Kumar.

RESULTS

For all limit cases where the fingers are either pointed or flat, the range of distance r for which the grasp is stable according the contact force model of Howard and Kumar is summarized in Tab. 1. The second column shows whether a finger is pointed (p) or flat (f). The third column contains the resulting r at infinitely stiff contact points. The last column shows r with a finite normal contact stiffness $k_{n,i}$ in each ith contact point.

For dynamic simulations in SPACAR, the stability of grasps with infinite stiffness at all contact points can not be determined. When one, two or three finite stiffness elements are applied, the stability results of the simulations agree to those obtained with the contact force model of Howard and Kumar (see Tab. 1, column 4). For case 4, these numerical results are summarized in Tab. 2. For the stability of the grasp it does not matter whether the finite stiffness is present in the finger or in the object.

DISCUSSION

The most important observation is the large difference in stability between grasping with pointed and flat finger tips. Table 1 shows that the stability even inverts when pointed finger tips become flat or vice versa. Distance r for which the grasp is stable ranges between $(-\infty, \infty)$, depending on the chosen finger geometry.

The large difference between pointed and flat finger tips is caused by the different character of the contact forces. This is

Table 1. Predicted stability as function of r for differently shaped fingers, derived with the method of Howard and Kumar.

	Finger	r [mm]	
	(1,2,3)	$k_{n,i} \to \infty$	$k_{n,i}$ finite
1	ppp	$>$-60	$>$ -60
2	fff	$<$-60	$<$ -60
3	ppf	< 60	$< 60 - \frac{F_{n0}}{3}\left(\frac{k_{n,1}+k_{n,2}+4k_{n,3}}{k_{n,1}k_{n,2}+k_{n,1}k_{n,3}+k_{n,2}k_{n,3}}\right)$
4*	ffp	> 60	$> 60 + \frac{F_{n0}}{3}\left(\frac{k_{n,1}+k_{n,2}+4k_{n,3}}{k_{n,1}k_{n,2}+k_{n,1}k_{n,3}+k_{n,2}k_{n,3}}\right)$
5	pfp	$> \ 0$	$> \ 0 + \frac{F_{n0}}{3}\left(\frac{k_{n,1}+4k_{n,2}+k_{n,3}}{k_{n,1}k_{n,2}+k_{n,1}k_{n,3}+k_{n,2}k_{n,3}}\right)$
6	fpf	$< \ 0$	$< \ 0 - \frac{F_{n0}}{3}\left(\frac{k_{n,1}+4k_{n,2}+k_{n,3}}{k_{n,1}k_{n,2}+k_{n,1}k_{n,3}+k_{n,2}k_{n,3}}\right)$
7	pff	$< \ 0$	$< \ 0 - \frac{F_{n0}}{3}\left(\frac{4k_{n,1}+k_{n,2}+k_{n,3}}{k_{n,1}k_{n,2}+k_{n,1}k_{n,3}+k_{n,2}k_{n,3}}\right)$
8	fpp	$> \ 0$	$> \ 0 + \frac{F_{n0}}{3}\left(\frac{4k_{n,1}+k_{n,2}+k_{n,3}}{k_{n,1}k_{n,2}+k_{n,1}k_{n,3}+k_{n,2}k_{n,3}}\right)$

(* Case 4 is equal to the initial grasp of Fig. 1)

Table 2. Predicted stability of Case 4 as a function of r for varying stiffness in the fingers, derived with the multibody dynamics simulation. (Stiffness in [Nmm^{-1}]).

	Finger 1 (flat)	Finger 2 (flat)	Finger 3 (pointed)	r [mm]
a	$k_{n,1} \to \infty$	$k_{n,2} \to \infty$	$k_{n,3} \to \infty$	N/A
b	$k_{n,1} \to \infty$	$k_{n,2} \to \infty$	$k_{n,3} = 30$	> 60
c	$k_{n,1} = 30$	$k_{n,2} \to \infty$	$k_{n,3} \to \infty$	> 60
d	$k_{n,1} = 30$	$k_{n,2} \to \infty$	$k_{n,3} = 30$	$> 60 + \varepsilon$
e	$k_{n,1} = 30$	$k_{n,2} = 30$	$k_{n,3} = 30$	$> 60 + 4\varepsilon$

$$\left(\varepsilon = \frac{1}{6}\frac{F_n}{k_n} \approx 0.0167\right)$$

illustrated in Fig. 6, where an object is grasped between two fingers. The grasp between pointed fingers is stable for small rotations, because the contact forces remain perpendicular to the object surface and generate a counteracting moment. Contrary, the grasp with flat finger tips is unstable.

A second remarkable observation is the major contribution of the contact geometry on the stability, relative to the local contact stiffness and preload. For case 4 with equal finite contact stiffnesses at all contact points, the grasp is stable when $r > 60 + 4\varepsilon$. The first term depends on geometry only, while ε comprises the local deformation with the order of magnitude $\varepsilon = O(F_{n0}/k_n)$. This ε is generally small compared to the characteristic length of

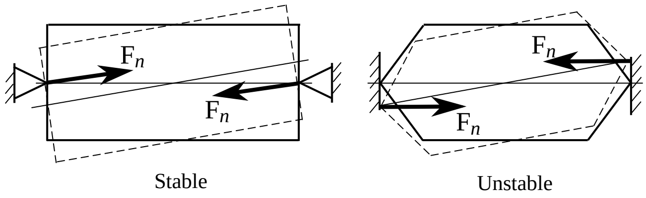

Figure 6. Grasping with pointed fingers (left) is stable, whereas with flat finger tips (right) is unstable.

the grasped object, except when the stiffness is very small compared to the preload. For case 1 and 2 with only pointed or flat fingers, respectively, the stability even does not depend on local deformations at all. The same is true for all cases when only one contact point has finite stiffness. Thus, the stability of these grasps is dominated by the contact geometry.

Finally, the obtained stability results might (again) completely change, when slightly different assumptions are used. Cutkosky and Wright [8] assume also tangential and rotational stiffness of the fingers, which also influences the character of the contact forces. Then, the opposite result is found: grasping with flat finger tips is more likely to be stable than grasping with pointed fingers. This underlines the message of this paper that the predicted object stability is indeed strongly dependent on the character of the contact forces, and the resemblance of the used models with the real grasp situation must thoroughly be investigated.

CONCLUSIONS

In this paper, the grasp stability was predicted for all limit cases of zero and infinite contact curvatures of a planar grasps with three fingers. In addition, the normal contact stiffness was varied from finite to infinite. The stability was predicted based on the contact force model of Howard and Kumar [1], and verified by a multibody dynamics simulation.

It was demonstrated that the predicted object stability is strongly dependent on the character of the contact forces. When rigid objects and fingers with only normal contact stiffness are assumed, then grasps with pointed fingers are more likely to be stable than grasps with flat finger tips. Furthermore, we showed that the grasp stability is dominated by the contact geometry, while local contact stiffness and preload only have a minor effect. Thus, from a designers viewpoint, pointed fingers are preferred, and local deformations need not be considered for design optimization based on the predicted grasp stability.

ACKNOWLEDGMENT

This work has been carried out as part of the FALCON project under the responsibility of the Embedded Systems Institute. This project is partially supported by the Netherlands Ministry of Economic Affairs under the Embedded Systems Institute (BSIK03021) program.

REFERENCES

[1] Howard, W. S., and Kumar, V., 1996. "On the stability of grasped objects". *IEEE Transactions on Robotics and Automation*, **12**(6), pp. 904–917.
[2] Baker, B. S., Fortune, S., and Gross, E., 1985. "Stable prehension with a multi-fingered hand". In Proceedings of the IEEE International Conference on Robotics and Automation, Vol. 2, pp. 570–575.
[3] Sinha, P. R., and Abel, J. M., 1992. "A contact stress model for multifingered grasps of rough objects". *IEEE Transactions on Robotics and Automation*, **8**(1), pp. 7–22.
[4] Herder, J. L., and Schwab, A. L., 2004. "On dynamically equivalent force systems and their application to the balancing of a broom or the stability of a shoe box - DETC2004-57188". In Proc. of DETC2004 ASME Design Engineering Technical Conferences.
[5] van der Werff, K., 1977. "Kinematic and dynamic analysis of mechanisms, a finite element approach". PhD thesis, Delft University of Technology, Delft, The Netherlands.
[6] Jonker, J. B., 1988. "A finite element dynamic analysis of flexible spatial mechanisms and manipulators". PhD thesis, Delft University of Technology, Delft, The Netherlands.
[7] Schwab, A. L., and Meijaard, J. P., 2003. "Dynamics of flexible multibody systems with non-holonomic constraints: a finite element approach". *Multibody System Dynamics*, **10**, pp. 107–123.
[8] Cutkosky, M. R., and Wright, P. K., 1986. "Friction, stability and the design of robotic fingers". *The International Journal of Robotics Research*, **5**(4), pp. 20–37.