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EXPERIMENTAL VALIDATION OF THE LATERAL DYNAMICS OF A BICYCLE ON A TREADMILL

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ABSTRACT

In this paper, an experimental validation of the lateral dynamics of a bicycle running on a treadmill is presented. From a theoretical point of view, bicycling straight ahead on a treadmill with constant belt velocity should be identical to bicycling on flat level ground with constant forward speed. However, two major differences remain: first, stiffnesses of the contact of the tire with the belt compared to the contact on flat level ground; second, the belt velocity is fixed with respect to the world, irrespective of the change in heading of the bicycle on the treadmill. The admissibility of these two differences is checked by comparing experimental results with numerical simulation results.

The numerical simulations are performed on a three-degree-of-freedom benchmarked bicycle model [1]. For the validation we consider the linearized equations of motion for small perturbations of the upright steady forward motion. This model has been validated experimentally in a previous work [2].

The experimental system consists of an instrumented bicycle without a rider on a large treadmill. Sensors are present for measuring the roll rate, yaw rate, steering angle, and rear wheel rotation. Measurements are recorded for the case in which the laterally perturbed bicycle coasts freely on the treadmill. From these measured data, eigenvalues are extracted by means of curve fitting. These eigenvalues are then compared with the results from the linearized equations of motion of the model. As a result, the model appeared to be accurate within the normal bicycling speed range, and in particular the transition from stable to unstable weave motion was very well predicted.

Keywords: Bicycle dynamics, experiments, instrumentation, treadmill, multibody dynamics, non-holonomic constraints.

1 Introduction

One of the characteristics of a bicycle is that it is highly unstable at low speed but easy to stabilize at moderate to high speed. Some bicycles can even show self-stability in the normal bicycling speed range. After the invention of the bicycle, more than 100 years ago [3], there has been a sudden revival in the research on the dynamics and control of a bicycle [4,1,5]. Results on the open loop stability are well established now [1], but little is know on how the rider controls the mostly unstable bicycle and what handling qualities are.

Recently a research program has been started at Delft University of Technology to investigate experimentally rider control during normal bicycling, model this behaviour and try to define the concept of handling qualities for bicycles. Instead of doing the experiments on the open road, there is the wish to execute the experiments in a more controlled environment. A large treadmill is such a controlled environment where one can look at rider control during normal straight-ahead bicycling or for small lat-

Figure 1. Large treadmill, courtesy of the faculty of Human Movement Sciences, Free University of Amsterdam, together with TUDelft instrumented bicycle.

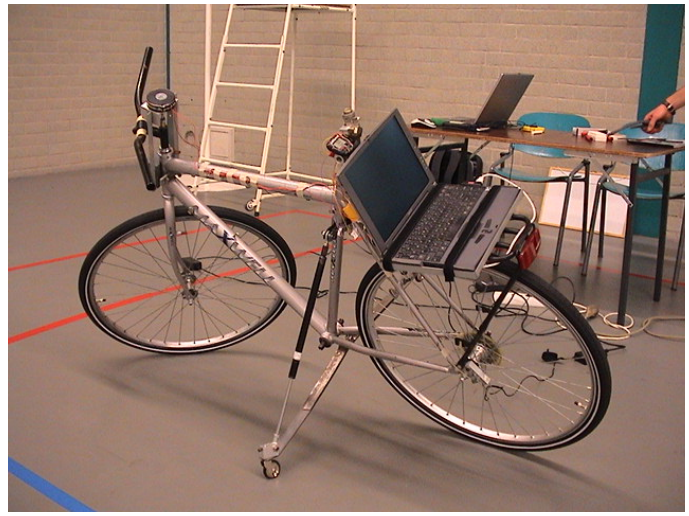

Figure 2. Instrumented bicycle from [2], with all the measurement equipment installed. Sensors are present for measuring the roll rate, yaw rate, steering angle, and rear wheel rotation. Data are collected via a USB-connected data acquisition unit on the laptop computer, mounted on the rear rack.

eral motions like lane change manoeuvres. But how close is this to bicycling on the open road?

One of the big problems with bicycling on a treadmill is the conflicting information which the rider gets. Although he is bicycling with respect to the moving belt he remains stationary with respect to the surrounding world. This is very confusing in the beginning. However, we now know from experience that after some time, riders can easily adapt to this awkward situation.

There remains the question of how good the treadmill mimics bicycling on flat level ground from a purely mechanical point of view. From a theoretical point of view, bicycling straight ahead on a treadmill with constant velocity should be identical to bicycling straight ahead with constant forward speed on flat level ground. However, there remain two problems. First, the different stiffness of the contact of the tire with the belt, and second, that the direction of the forward velocity is fixed with respect to the world irrespective of the change in heading of the bicycle.

This paper investigates the validity of bicycling on a treadmill by comparing the lateral motions of an instrumented riderless bicycle [2] with results from a three-degree-of-freedom benchmarked bicycle model [1], which has been experimentally validated in [2]. The experimental system consists of an instrumented bicycle without rider on a large treadmill, see Figure 1. On the bicycle, sensors are present for measuring the roll rate, yaw rate, steering angle, and rear wheel rotation, see Figure 2. Trainer wheels prevent the complete fall of the bicycle for unstable conditions. Measurements are recorded for the case in which the bicycle coasts freely on the treadmill surface after some small lateral perturbation which initiates the lateral motion. From these measured data, eigenvalues are extracted by means of curve fitting. These eigenvalues are then compared with the results from the linearized equations of motion of the model.

The organization of the paper is as follows. After this introduction, the treadmill, instrumented bicycle, and linearized equations of motion are described. Then the test procedure and a comparison of the experimental and numerical results are presented and discussed. The paper ends with some conclusions.

2 Treadmill and Instrumented Bicycle

The treadmill, see Figure 1, has a usable belt surface of 3×5 m which can be inclined from -5 to 15 deg, and a regulated maximum speed of 35 km/h. An emergency stop can stop the belt within 1 sec. The surface of the treadmill belt is of the ordinary rubber-like structure with moderate roughness. The treadmill is manufactured by Forcelink B.V., The Netherlands, and stationed at the faculty of Human Movement Sciences, Free University of Amsterdam.

The instrumented bicycle, see Figure 2, used in the test is fully described in [2]. It is a standard city-bicycle where all the superfluous parts of the bicycle are removed. Sensors are present for measuring the roll rate, yaw rate, steering angle and rear wheel rotation. The data are collected on a laptop computer mounted on the rear rack. Trainer wheels prevent the complete fall of the bicycle for unstable conditions.

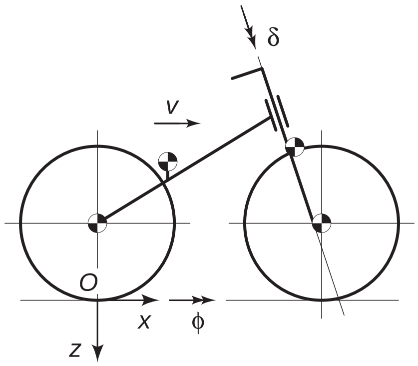

Figure 3. The bicycle model: four rigid bodies (rear wheel, rear frame, front handlebar assembly, front wheel) connected by three revolute joints (rear hub, steering axis, front hub), together with the coordinate system, and the degrees of freedom.

3 Linearized equations of motion for the bicycle model

The bicycle model used is the so-called Whipple [6] model which recently has been benchmarked [1]. The model, see Figure 3, consists of four rigid bodies connected by revolute joints. The contact between the knife-edge wheels and the flat level surface is modelled by holonomic constraints in the normal direction and by non-holonomic constraints in the longitudinal and lateral direction. In the absence of a rider (or with a rider rigidly attached to the rear frame) we assume no-hands operation. The resulting non-holonomic mechanical model has three velocity degrees of freedom: forward speed v, lean rate $\dot{\phi}$ and steering rate $\dot{\delta}$.

For the comparison we consider the linearized equations of motion for small perturbations of the upright steady forward motion, which are fully described in [1]. They are expressed in terms of small changes in the lateral degrees of freedom (the rear frame roll angle, ϕ, and the steering angle, δ) from the upright straight ahead configuration $(\phi,\delta) = (0,0)$, at a forward speed v, and have the form

$$\mathbf{M}\ddot{\mathbf{q}} + v\mathbf{C_1}\dot{\mathbf{q}} + [\mathbf{K_0} + v^2\mathbf{K_2}]\mathbf{q} = \mathbf{f}, \quad (1)$$

where the time-varying variables are $\mathbf{q} = [\phi, \delta]^T$ and the lean and steering torques $\mathbf{f} = [T_\phi, T_\delta]^T$. The coefficients in this equation are: a constant symmetric mass matrix, $\mathbf{M}$, a damping-like (there is no real damping) matrix, $v\mathbf{C_1}$, which is linear in the forward speed v, and a stiffness matrix which is the sum of a constant symmetric part, $\mathbf{K_0}$, and a part, $v^2\mathbf{K_2}$, which is quadratic in the forward speed. The forces on the right-hand side, $\mathbf{f}$, are the applied forces which are energetically dual to the degrees of freedom $\mathbf{q}$.

The entries in the constant coefficient matrices $\mathbf{M}, \mathbf{C_1}, \mathbf{K_0}$, and $\mathbf{K_2}$ can be calculated from a non-minimal set of 25 bicycle parameters as described in [1]. The procedure and measured values of these parameters for the instrumented bicycle can be found in [2]. From these measured parameters the coefficient matrices of the linearized equations of motion are calculated as:

$$\mathbf{M} = \begin{bmatrix} 7.98981, & 0.89569 \\ 0.89569, & 0.29857 \end{bmatrix}, \mathbf{C_1} = \begin{bmatrix} 0 & , & 7.17025 \\ -0.59389, & & 1.32610 \end{bmatrix},$$
$$\mathbf{K_0} = \begin{bmatrix} -109.91168, & -13.45745 \\ -13.45745, & -4.82272 \end{bmatrix}, \mathbf{K_2} = \begin{bmatrix} 0 & , 11.19798 \\ 0 & , 1.42200 \end{bmatrix}. \quad (2)$$

Then, with these coefficient matrices the characteristic equation,

$$\det\left(\mathbf{M}\lambda^2 + v\mathbf{C_1}\lambda + \mathbf{K_0} + v^2\mathbf{K_2}\right) = 0, \quad (3)$$

can be formed and the eigenvalues, λ, can be calculated. These eigenvalues, in the forward speed range of $0 \le v \le 10$ m/s, are presented by the continuous lines in Figure 6. In principle there are up to four eigenmodes, where oscillatory eigenmodes come in pairs. Two are significant and are traditionally called the *capsize mode* and *weave mode*. The capsize mode corresponds to a real eigenvalue with eigenvector dominated by lean: when unstable, the bicycle just falls over like a capsizing ship. The weave mode is an oscillatory motion in which the bicycle sways about the headed direction. The third remaining eigenmode is the overall stable *castering mode* which corresponds to a large negative real eigenvalue with eigenvector dominated by steering.

4 Experimental Procedure and Results

Measurements were recorded for the case in which the laterally perturbed bicycle coasted freely on the treadmill. From these measured data eigenvalues were extracted by means of curve fitting. These eigenvalues were then compared with the results from the linearized equations of motion of the model. As a result, the model appeared to be fairly accurate for the normal bicycling speed range.

4.1 Expected motions

Looking at the eigenvalue plot, Figure 6, the following bicycle motions during the experiments can be expected. At low speed the motion of the free-coasting laterally-perturbed bicycle will be dominated by the unstable weave motion. Both the capsize and the castering modes are very stable here and any initial transient will quickly die out. The time frame for measurement will be short due to the unstable nature of the weave motion. Then in the stable speed range, again the motion will be dominated by the oscillatory weave motion. The moderately stable/unstable capsize motion will only give a small offset in the lean rate. Here, the measurement window will be large since the oscillatory weave motion is stable.

4.2 Test procedure

The experiments were carried out on the large treadmill of the faculty of Human Movement Sciences at the Free University of Amsterdam. The 3×5 m usable belt surface has a rubber-like structure with moderate roughness.

A total of 88 runs were carried out within a belt speed range of 10 to 30 km/h (2.8 to 8.3 m/s). In each run the bicycle was first put manually in the vertical equilibrium position and given some time for the wheels to speed up and get into the steady upright motion for the given belt speed. The the bicycle was released and caught before the fall.

To measure the dynamic response of the bicycle at different speeds and to calculate the corresponding motion eigenvalues, the bicycle had to show some lateral dynamics. At speeds below the stable speed range no external excitation was required. Due to small imperfect or non perfect initial conditions the bicycle always started to weave about its general heading and this motion was measured. The time window for measuring was short due to the unstable motion.

For runs in the stable speed range the bicycle set itself in an upright position and showed no dynamic behaviour unless it was given a lateral excitation. This excitation was accomplished by applying a lateral impulse to the bicycle by simply hitting the bicycle's rear frame by hand in the lateral direction at approximately the insertion of the saddle pillar with the down tube. A side effect of this perturbation was that after the stable weave oscillation had died-out, the bicycle was heading in a slightly different direction and slowly running off the belt.

4.3 Stored data

The frequency of the weave of motion is low, of the order of 1 Hz (see Figure 6) and therefore only a low sample rate was needed here. However, the measurement of the forward speed by means of the 10 magnet ring needed a higher sampling rate. The first tests were measured with a 100 Hz sample frequency. Then to ensure no aliasing in the speedometer signal would take, 500 Hz was used. Unfortunately higher sampling frequencies gave a very erratic steering angle potentiometer signal at the recorder. The recorded data for each run were stored in a text file.

Every run was also recorded on video. Examples of these recordings can be found at [7]. This turned out to be essential for the processing of the run data and helped to identify nonstandard measurements, the quality of the launch, etc. It was thus possible to compare the recorded data afterwards with the video images and to extract the relevant data for the calculation of the eigenvalues from each file.

4.4 Data analysis

For each run the raw data were transferred to Matlab and at first inspected visually. A plot of the raw data for run 1202 is shown in Figure 4. In the figure the signals from top to bottom are: battery voltage, steering angle, lean rate, yaw rate, and speedometer. These graphs were used, together with the videos of the runs, to locate the time window in which the bicycle coasted freely for each run. Once manually located, a curve fit of the time histories of the eigenmotions was performed on the lean rate data to extract the measured eigenvalues, see Figure 5.

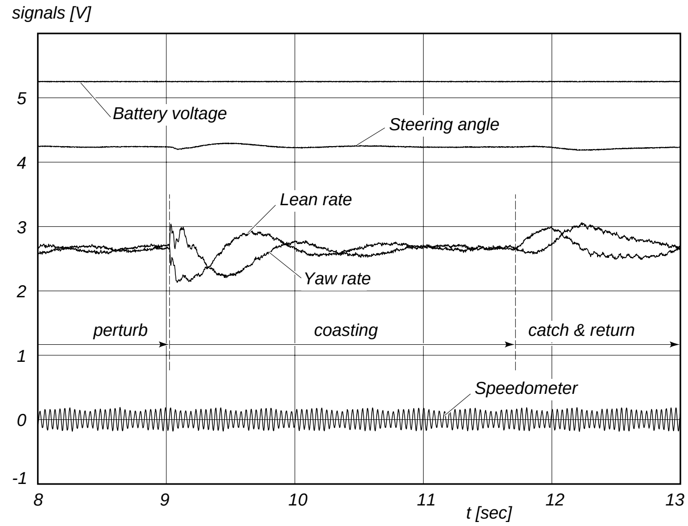

Figure 4. The raw measured data from run 1202. The signals from top to bottom are: battery voltage, steering angle, lean rate, yaw rate, and speedometer. The forward speed is around 5.5 m/s, which is clearly within the stable speed range (see Figure 6). Note the three different motion regimes: perturb, coasting, and catch & return.

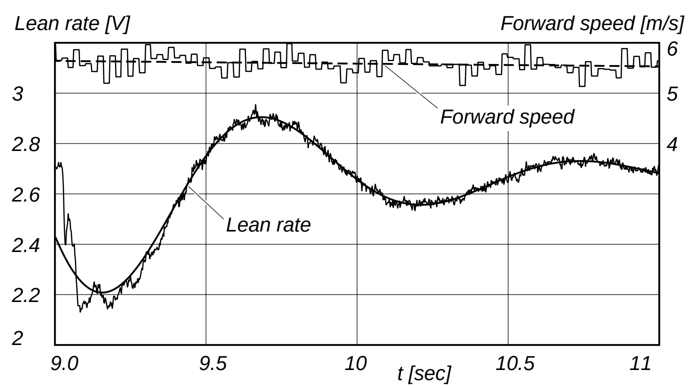

Figure 5. Least-squares curve fit of the oscillatory stable lean rate time history (solid smooth curve) to the measured lean rate (ragged line) for run 1202, together with the measured forward speed (staircase line) and linear regression of the forward speed (dashed line). Note the slight decrease in forward speed (from 5.6 to 5.5 m/s) during the measurement. The extracted weave eigenvalue within the time window of $9.4 \leq t \leq 11.0$ sec is $\lambda_{weave} = -1.32 \pm 5.96i$ 1/s.

Above a forward speed of 0.1 m/s there is in principle a sum of three eigenmodes to be fitted: the castering mode, the capsize mode and the oscillatory weave mode, see Figure 6. The castering mode is highly damped and will vanish quickly from the transient signal. The capsize mode is also reasonably damped below the weave speed and is mildly unstable above the capsize speed resulting here in a small and slow lean-rate offset. Initially we tried to fit the sum of the capsize mode and the oscillatory weave mode to the lean rate data but it turned out that the contribution of the capsize mode was very small. This resulted into almost random values for the capsize eigenvalue, which can be explained as follows. Below the weave speed, the capsize mode is well damped and vanishes quickly, whereas the weave mode is unstable and will dominate the response. Above the weave speed, the capsize eigenvalue is small compared to the weave eigenvalue, in an absolute sense, and again the weave mode will dominate the response. Therefore only an exponentially damped or growing oscillatory weave motion was fitted to the data. The function to be fitted to the measured lean rate was taken as

$$\dot{\phi} = c_1 + e^{dt}[c_2\cos(\omega t) + c_3\sin(\omega t)], \tag{4}$$

with the weave frequency $\omega = \mathrm{Im}(\lambda_{weave})$, the weave damping $d = \mathrm{Re}(\lambda_{weave})$ and the three constants: c_1 for the offset, c_2 for the cosine amplitude and c_3 for the sine amplitude. Since the weave frequency and damping appear in a non-linear way in the function a non-linear least-squares fitting method was used (Matlab's `fminsearch`) to extract the eigenvalues.

The speedometer signal, see Figure 4, was an oscillatory signal with a frequency of ten times the rear wheel rotation frequency. The signal was converted to a forward speed by counting the time between successive zero crossings. As each crossing represents a $1/20$th of a complete rear wheel rotation an average speed for that portion could be calculated; this is the staircase line in Figure 5. As the forward speed during the coasting section of the measurements slowly decreased due rolling resistance, a speed range was assigned to the calculated λ's instead of one specific speed. This speed range was calculated by looking at the linear regression of the speed for the chosen time window, see Figure 5.

Finally, for all runs, in Figure 6, the measured eigenvalues were plotted on top of the calculated eigenvalues where horizontal bars are used to indicate the forward speed variation during the measurements.

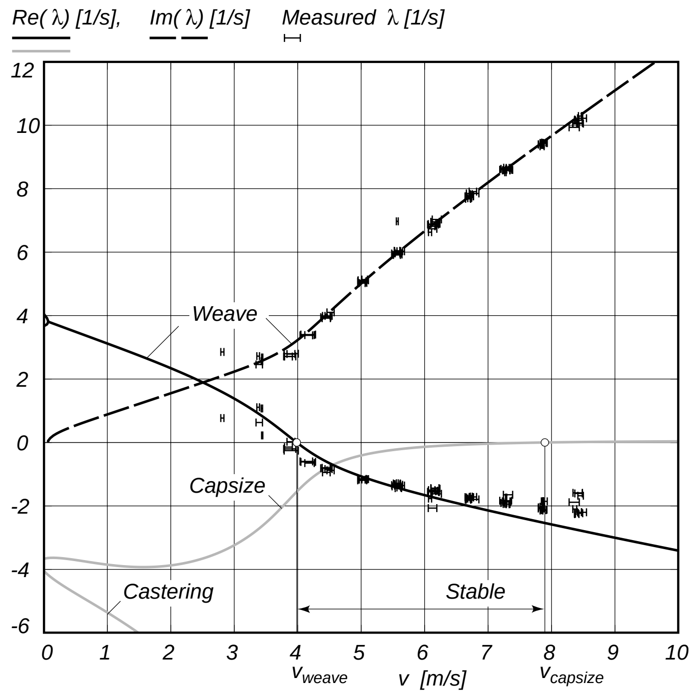

Figure 6. Measured eigenvalues λ (horizontal bars) and calculated eigenvalues λ (continuous lines) for the instrumented bicycle on the treadmill, from Figures 1 and 2, in the forward speed range of $0 \leq v \leq 10$ m/s. For the measured values only the weave motion is considered. The lengths of the horizontal bars indicate the forward speed range during the measurement. For the calculated values the solid lines correspond to the real part of the eigenvalues and the dashed line corresponds to the imaginary part of the eigenvalues. The zero crossings of the real part of the eigenvalues are for the weave motion at the weave speed $v_{\mathrm{weave}} \approx 4.0$ m/s and for the capsize motion at capsize speed $v_{\mathrm{capsize}} \approx 7.9$ m/s. The speed range for asymptotic stability of the instrumented bicycle is $v_{\mathrm{weave}} < v < v_{\mathrm{capsize}}$.

4.5 Discussion

At speeds above 3 m/s, the predicted weave frequency and damping by the model were forecasted accurately. The transition from the unstable to the stable region around the weave speed is accurately described by the model.

In the unstable speed region, in particular below 3 m/s, it turned out to be very difficult to measure the motion of the bicycle. The time window for measurement was very short compared with the period of the weave motion. Therefore, trying to fit only a part of a harmonic function to the measured data turned out to be very difficult and the results showed considerable spread.

The yaw rate signal was of the same quality as the lean rate signal, but the steering angle signal turned out to be too small and too erratic, due to noise in the potentiometer, to use.

5 Conclusions

The experimental results show a good agreement with the results obtained by a linearized analysis on a three-degree-of-freedom dynamic model of an uncontrolled bicycle. The transition from stable to unstable speeds is also well predicted. This shows that the tire-belt compliance and tire-belt slip, and the small changes in bicycle heading relative to the belt velocity are not important for the lateral dynamics of the bicycle on a treadmill.

Therefore we conclude that riding a bicycle on a treadmill with constant belt velocity is dynamically equivalent to riding a bicycle on flat level ground around the straight ahead direction with constant speed.

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