FRANCIS JOHN WELSH WHIPPLE

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Francis John Welsh Whipple, who died on 25 September, 1943, at the age of 67, was known to this Society mainly for his series of papers on generalized hypergeometric series. These papers were probably the most important part of his mathematical work, but they form only one section of his scientific activity.

Whipple was born on 17 March, 1876. His father was George Mathews Whipple, who was Superintendent of Kew Observatory from 1878 to 1893, and his mother was Martha Beckley, whose father, Robert Beckley, had been on the staff of Kew Observatory in the eighteen-fifties and had designed the famous Kew self-recording meteorological instruments.

He was educated at Merchant Taylors' School, and later at Trinity College, Cambridge, where he was a Scholar. In 1897 he graduated as Second Wrangler, and the following year was placed in Class I, Division I of Part II of the Mathematical Tripos; in 1899 his Smith's Prize essay received honourable mention. He was awarded the degree of Sc.D. at Cambridge in 1929.

For thirteen years after leaving Cambridge (1899–1912) Whipple was at Merchant Taylors' School as mathematical master, and then he joined the Meteorological Office, working at the headquarters office in London, first as superintendent of the Instruments Division and then of the Climatology Division. In 1925 he succeeded Dr. Charles Chree as Superintendent of Kew Observatory, where his father and his maternal grandfather had served before him, and he retired from this position in 1939.

Whipple had married, in 1911, Violet Mary Josephine, daughter of Sir Thomas J. Pittar, K.C.B., and they had one son. Mrs. Whipple predeceased her husband in 1926, her death being the result of a motor accident.

Whipple's main mathematical work was concerned with generalized hypergeometric series with argument 1 or -1, and with the exception of 22 all his papers on this subject were published by this Society. His first interest in the subject became apparent in 1910 when he wrote paper 22 in conjunction with M. J. M. Hill. In this paper the main result obtained is that, if $H\begin{pmatrix}\alpha, & \beta \\ \gamma, & \delta\end{pmatrix}$ denotes the series

$$\frac{1}{(\gamma-1)(\delta-1)}+\frac{\alpha\beta}{(\gamma-1)\gamma(\delta-1)\delta}+\frac{\alpha(\alpha+1)\beta(\beta+1)}{(\gamma-1)\gamma(\gamma+1)(\delta-1)\delta(\delta+1)}+\dots,$$

then
$$H\binom{\alpha,\ \beta}{\gamma,\ \delta} = H\binom{\gamma-\alpha,\quad \gamma-\beta}{\gamma,\quad \gamma+\delta-\alpha-\beta}.$$

Evidently, in the notation of generalized hypergeometric series,

$$H\binom{\alpha,\ \beta}{\gamma,\ \delta} = \frac{1}{(\gamma-1)(\delta-1)}\,{}_3F_2\left[\begin{matrix}1,\ \alpha,\ \beta;\\ \gamma,\ \delta\end{matrix}\right],$$

where the argument of the series on the right is 1. There are also results connecting series of the type

$$H\binom{a,\ b,\ c}{l,\ m,\ n},$$

though here the relations connect three series instead of two. Actually the main result of this paper had been given by Saalschütz many years before, but the paper is interesting because it probably had a considerable influence on Whipple's later work. It is also interesting for the fact that the results are used to transform some slowly convergent series into rapidly convergent series, which perhaps forms a slight link between this subject and the more practical subjects in which Whipple was interested.

Whipple did not return to hypergeometric series until 1923. In this year Hardy published his paper "A chapter from Ramanujan's note-book"*, which, besides giving an account of Ramanujan's work on hypergeometric series, collected together the known results, which up to that time were somewhat scattered. There is no doubt that this paper by Hardy, and the papers which Whipple wrote from 1923 onwards, were the inspiration which led to numerous papers by other writers on this subject. In his paper Hardy acknowledges the use of a manuscript by Whipple. This was 4, which is devoted to a discussion of relations connecting series of the type ${}_3F_2$ with unit argument. These had been discussed in great detail as long ago as 1879 by Thomae, who had obtained a mass of formulae connecting two or three series of the type ${}_3F_2$. Whipple's object in 4 is to "provide a simple clue to the labyrinth", and in this he is remarkably successful. With the notation which he introduces, combined with his tables of parameters, it is a comparatively simple matter to find any formula one wants connecting two or three series ${}_3F_2$, or to recognise any formula of this kind as a known result. Perhaps it is in this "bringing to heel" of masses of formulae that Whipple shows his powers most. In this paper, for example, he is dealing with a set

* *Proc. Camb. Phil. Soc.*, 21 (1923), 492–503.

of 120 series of the type ${}_3F_2$, and one can readily imagine how easy it would be to get lost in a large number of formulae. It was in this paper that he obtained his well-known sum for a particular ${}_3F_2$.

Paper **5** was the first paper that Whipple wrote on a class of series which he called "well-poised". This was the name he gave to the series

$$F\begin{bmatrix} a, & b, & c, & d, & e, & \ldots ; & x \\ & p, & q, & r, & s, & \ldots & \end{bmatrix},$$

where $a+1=b+p=c+q=\ldots$.

Shortly before this paper was written, Watson had given a simple proof of Dixon's theorem for the sum of a well-poised ${}_3F_2$ of argument 1, which he deduced from Kummer's theorem for the sum of a well-poised ${}_2F_1$ with argument -1. Whipple used a similar argument to deduce from Dixon's theorem a transformation of a well-poised ${}_4F_3\,(-1)$, and he proceeded in this way, step by step, until he finally arrived at a transformation of a well-poised ${}_7F_6(1)$ into a ${}_4F_3(1)$. I think it is correct to say that from this formula could be derived every previously known result concerning generalized hypergeometric series with argument 1 or -1. It is easy, for example, to deduce Dougall's theorem for the sum of a certain well-poised ${}_7F_6$, which was the fundamental formula of Hardy's paper already quoted. Whipple obtained from his transformation a number of new formulae giving sums of various well-poised series of lower order. He also showed that, as a limiting case, his transformation gave a well-poised ${}_6F_5(-1)$ in terms of a ${}_3F_2(1)$, and conversely it expressed any ${}_3F_2(1)$ as a well-poised ${}_6F_5(-1)$. Since he had already worked out the relations connecting various series of the type ${}_3F_2(1)$ in **4**, he was now in a position to obtain many transformations of well-poised ${}_6F_5(-1)$ into series of the same type or into various series ${}_3F_2(1)$, and this he did in the same paper. He also noticed that his most general transformation in **5** gave a well-poised ${}_7F_6(1)$ in terms of a "Saalschützian" series ${}_4F_3(1)$, that is one in which the denominator parameters have a sum greater than that of the numerator parameters by 1, this series also terminating. Conversely, any Saalschützian ${}_4F_3(1)$ could be transformed into a well-poised ${}_7F_6(1)$ in various ways, and by repeated use of the transformation he could find a large number of relations connecting well-poised ${}_7F_6$ and Saalschützian ${}_4F_3$. These relations were worked out with his usual skill in **6**, where he gave formulae connecting 60 non-terminating well-poised series, 10 Saalschützian series, and 15 terminating well-poised series.

In Whipple's transformation the Saalschützian ${}_4F_3$ was subject to the restriction that it must terminate. This meant that in the well-poised

${}_7F_6$ one parameter, or a linear combination of the parameters, had to be a negative integer. Some years after Whipple's paper was published I noticed that relations existed connecting two-well poised series ${}_7F_6$ in which there was no numerical restriction of this kind on the parameters, but my formulae did not generalize all those given by Whipple. It did not occur to me how to generalize the other formulae, but apparently Whipple realised, as soon as he read my paper, that he must look for relations connecting three well-poised ${}_7F_6$, corresponding exactly to the relations connecting three series ${}_3F_2$ given in **4**. After generously waiting for me to publish a paper on the subject, and finding that no such paper was forthcoming, he wrote his paper **10**, in which he gave the formulae connecting three well-poised ${}_7F_6$, these series not being numerically restricted. He also, as usual, introduced a new notation, by means of which he considered the relations between members of a group of 192 well-poised series ${}_7F_6$; and he also gave relations between members of an allied group of 160 non-terminating Saalschützian ${}_4F_3$.

A few years before this paper was written I had discovered two formulae each connecting four non-terminating well-poised series ${}_9F_8$. If one of the parameters is a negative integer, these relations reduce to formulae connecting two or three well-poised ${}_9F_8$, and by making the negative integer tend to $-\infty$ we can obtain relations connecting two or three unrestricted well-poised ${}_7F_6$. Thus, without realising it, I had provided the means of working out all Whipple's formulae in **10**; but I consider that it was fortunate that the work fell to Whipple, who could deal with large numbers of formulae in a far better way than I could ever have done.

At this time it seemed that everything possible had been discovered about well-poised series, but this was not so. In 1936 Whipple wrote his final paper (**11**) on the subject. In this paper he finds transformations for trigonometric series whose coefficients are terms of well-poised hypergeometric series with unit argument. Particular cases of these results yield relations connecting three well-poised ${}_7F_6$, or four well-poised ${}_9F_8$, and so on. In my relations connecting four ${}_9F_8$ the sum of the denominator parameters has to exceed that of the numerator parameters by two. In Whipple's relation connecting four ${}_9F_8$ there is no such restriction.

It is perhaps strange that Whipple never did any work on basic hypergeometric series, but his influence is evident in this field, since many of the results true for well-poised series of the ordinary type have their analogues for well-poised basic series.

Whipple's work on well-poised series extended over about a dozen years, but during this time he was interested in other aspects of hypergeometric series. Thus in **7** he considered series which he termed "nearly-poised", that is series of the type

$$F\begin{bmatrix} a, & b, & c, & \dots; & x \\ & \kappa-b, & \kappa-c, & \dots & \end{bmatrix}$$

in which pairs of parameters have the same sum except for the first pair (counting 1 as a denominator parameter). From his results he deduced corresponding formulae when the breakdown in the equality of sums occurs with the last pair. The most general transformation that he obtained was one for a nearly-poised series ${}_4F_3(1)$. He used his transformation of a nearly-poised ${}_3F_2(1)$ to give, in **15**, an extremely simple proof of some well-known results due to Cayley and Orr concerning the products of certain ordinary hypergeometric series.

In his paper **7** on nearly-poised series, Whipple had obtained a transformation of the general ${}_2F_1(-1)$ into a ${}_3F_2(1)$, a general ${}_2F_1$ being, of course, the simplest type of nearly-poised series. This led him to consider the series allied to the series ${}_2F_1(-1)$. In his paper **9** he obtains transformations of such a series into series of the type ${}_3F_2(1)$ and ${}_6F_5(-1)$, and another set in which the series are of the type ${}_4F_3(1)$.

Apart from his work on hypergeometric series, Whipple wrote a number of papers on other mathematical subjects, but these were more in the nature of isolated problems. Probably the best known result in these papers is given in **3**. This is the formula

$$e^{-mi\pi} Q_n{}^m(\cosh\alpha) = \sqrt{\left(\frac{\pi}{2}\right)} \frac{\Pi(m+n)}{\sqrt{(\sinh\alpha)}} P_{-m-\frac{1}{2}}^{-n-\frac{1}{2}}(\coth\alpha),$$

usually known as "Whipple's transformation of Legendre functions". It was suggested by certain integrals in **2**, in which the problem was the determination of Green's function for a wedge. This is typical of that part of Whipple's work which is not connected with hypergeometric series. It is, roughly, evenly divided between pure and applied mathematics, with a bias towards applied. Even in paper **1**, which is essentially on a subject in pure mathematics, he illustrates his results by considering the distribution of electricity on the surface of an insulated bowl. It is interesting to note that his first paper (**20**), written when he was a Scholar of Trinity, was an elaborate discussion on "The stability of the motion of a bicycle".

Apart from his work in mathematics Whipple did a great deal of work in other subjects. With regard to this I cannot do better than quote Sir George C. Simpson's notice in *Nature**. He says: "That Whipple

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* 13 Nov. 1943.

had outstanding ability as a mathematician his university career had given evidence, and throughout his life he maintained his interest in mathematics, but he was also a physicist of no mean order, and the physical problems of meteorology, or rather geophysics, were entirely congenial to him. Whipple was not by nature original, but if in his reading or in conversation he came across a mathematical or physical problem, he would devote time and energy to its solution. In this way seismology, atmospheric electricity and meteorological optics, with their innumerable concrete problems, interested him immensely, and he became an authority on all these subjects. It was the exhibition by Sir Napier Shaw, at the British Association meeting in Dublin, of a diagram showing unexplained pressure waves on a microbarograph which led to his outstanding paper on the great Siberian meteor, and it was the audibility in England of gunfire on the Western Front during the war of 1914–18 which led directly to his major work on the transmission of sound, which has given us so much information about the temperature of the upper atmosphere. Practically the whole of Whipple's numerous publications deal with problems of this nature ".

Quoting further from the same notice: " Whipple served on the National Committee for Geodesy and Geophysics of the Royal Society, and was one of the British delegates to the International Geophysical and Geodetic Conference which met in Washington in September, 1939. For a number of years he was chairman of the Seismological Committee of the British Association. He served on the Council of the Royal Meteorological Society and was president in 1937 and 1938. From 1930 until 1932 he was a member of the Board of the Institute of Physics ".

From all this it will be seen that Whipple's work on hypergeometric series was only a small part of his scientific work, but to some of us it was an intensely interesting part. I well remember how a somewhat lackadaisical interest was aroused in me as he published one paper after another, until a more vital interest gripped me as he developed the subject. For several years we corresponded on mathematical subjects before we met, and when we did eventually meet, through the kind thought and hospitality of Dr. Stoneley, I felt that I was meeting an old friend.

Perhaps Whipple's work on hypergeometric series appealed only to a fairly small number of mathematicians—though I think that it aroused some curiosity in the minds of many—but to those who worked in this field, and perhaps particularly to me, his influence was great and his work an inspiration.

Mathematical papers.

Proc. London Math. Soc. (2).

1. "On the behaviour at the poles of a series of Legendre's functions representing a function with infinite discontinuities", 8 (1910), 213–222.
2. "Diffraction by a wedge and kindred problems", 16 (1917), 94–111.
3. "A symmetrical relation between Legendre's functions with parameters $\cosh \alpha$ and $\coth \alpha$", 16 (1917), 301–314.
4. "A group of generalized hypergeometric series; relations between 120 allied series of the type $F\binom{a,\ b,\ c}{e,\ f}$", 23 (1925), 104–114.
5. "On well-poised series, generalized hypergeometric series having parameters in pairs, each pair with the same sum", 24 (1926), 247–263.
6. "Well-poised series and other generalized hypergeometric series", 25 (1926), 525–544.
7. "Some transformations of generalized hypergeometric series", 26 (1927), 257–272.
8. "On a theorem due to F. S. Macaulay, concerning the enumeration of power products", 28 (1928), 431–437.
9. "On series allied to the hypergeometric series with argument -1", 30 (1930), 81–94.
10. "Relations between well-poised hypergeometric series of the type ${}_7F_6$", 40 (1936), 336–344.
11. "Well-poised hypergeometric series and cognate trigonometric series", 42 (1937), 410–421.

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12. "The relation between the distributions of potential in the neighbourhood of a cylindrical conductor when it is charged and when it is placed in a uniform field of force", May 15, 1924, lv–lvii.

Journal London Math. Soc.

13. "A fundamental relation between generalized hypergeometric series", 1 (1926), 138–145.
14. "Algebraic proofs of the theorems of Cayley and Orr concerning the products of certain hypergeometric series", 2 (1927), 85–90.
15. "On a formula implied in Orr's theorems concerning the products of hypergeometric series", 4 (1929), 48–50.
16. "The sum of the coefficients of a hypergeometric series", 5 (1930), 192.
17. "On an integral formula for certain Legendre functions connected with the anchor ring", 7 (1932), 141–142.
18. "On transformations of terminating well-poised hypergeometric series of the type ${}_9F_8$", 9 (1934), 137–140.

Proc. Royal Society (A).

19. "Equal parallel cylindrical conductors in electrical problems", 96 (1920), 465–475.

Quarterly Journal of Pure and Applied Mathematics.

20. "The stability of the motion of a bicycle", 30 (1899), 312–348.
21. "On Lagrange's and other theorems and on the solution of equations by logarithmic series", 40 (1909), 368–373.
22. (With M. J. M. Hill.) "A reciprocal relation between generalized hypergeometric series", 41 (1910), 128–135.

Philosophical Magazine.

23. "The motion of a particle on a smooth rotating globe" (6), 33 (1917), 457–471.
24. "Diffraction of plane waves by a screen bounded by a straight edge" (6), 36 (1918), 420–424.
25. "The disturbance of the uniform temperature of the stratosphere by the vertical displacement associated with horizontal motion governed by the 'Geostrophic Law'" (6), 45 (1923), 778–782.
26. "On the best linear relation connecting three variables" (7), 1 (1926), 378–384.

Proc. Royal Soc. of Edinburgh.

27. (With W. H. McCrea.) "Random paths in two and three dimensions", 60 (1940), 281-298.

Apart from the above papers Whipple published about sixty papers in *Gerlands Beiträge zur Geophysik*, *Proceedings of the Physical Society*, *Quarterly Journal of the Royal Meteorological Society*, *Monthly Notices* and *Geophysical Supplement of the Royal Astronomical Society*, *Journal of Scientific Instruments*, *Journal of State Medicine*, *Meteorological Magazine*, *Geophysical Memoirs*, *Aeronautical Research Committee Reports*, *Transactions of the Faraday Society*, *Terrestrial Magnetism*, *Bulletin of the International Meteorological Association*, *Discovery*, and *Nature*, as well as numerous notes and articles in the *Math. Gazette*.

[I should like to express my gratitude to the Editor of *Nature* and to Sir George C. Simpson for their generosity in allowing me to make use of Sir George Simpson's notice in *Nature*. Apart from the quotations, I have obtained many other details from this notice. I am also indebted to Dr. Whipple's son, Mr. R. T. P. Whipple, for kindly providing me with a complete list of Dr. Whipple's publications, and to Mr. F. P. White, who helped me in various ways.]